Reliability Prediction Modelling for Wireless Communication Networks

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***Abstract-* Wireless Communication Networks reliability model is analysed in the given paper for studying and evaluating data transmission through unreliable wireless channel, subjected to distortions on the physical layer. The given model's states are defined by the different kinds of time between neighbouring failures, which is distributed according to Erlang ratio. The method of enhance of reliability of transmission through unreliable wireless channel (WCH) is suggested and tackled through in depth mathematical modelling.**

Keywords- *wireless communication channel; Reliability; Erlang distribution; erroneous packets.*

I. Introduction

The Physical Layer analysis of any Computer Communication network is very important. This is because many different problems concerning network execution and utilization are caused by errors and failures on the Physical Layer. Wireless communication networks are not the exception. Moreover, the problem of achieving high reliability and error tolerance is urgent in modern communication technologies such as mobile Internet Protocol (IP) or General Packet Radio Service (GPRS) networks [1].

A number of problems arise in a wireless communication channel context. These problems are caused by the following reasons. The first reason is related to non-reliability of the radio channel. In fact, a sufficiently high Bit Error Rate (BER) and very high synchronization failure probability as opposed to the qualitative wire and especially fiber optic channels characterize these channels. BER may be rather within $10^{-1} - 10^{-3}$. The second reason is that BER in the radio channel is not a constant value. As soon as the reliability of the wireless channel depends on several external reasons, BER is essentially a time function. However, in recent years there has been an intensive interest in wireless channels. A number of papers devoted to the impact of burst errors on the network reliability and on packet [2, 3].

II. Determining Probabilistic Model of the Wireless Channels

IEEE 802.11 provides a CSMA/CA (carrier sense multiple access with collision avoid) -based mechanism to allow nodes access wireless medium, with extensions to allow for the exchange of RTS/CTS (request-to-send/clear-to-send) handshake packets between the transmitter and the receiver. The RTS/CTS exchange is used to reserve a *transmission floor* for the subsequent data packet transmission. Nodes transmit their control and data packets at a fixed (maximum) power level, preventing all other potentially interfering nodes from starting their own transmissions. Any node that hears the RTS or the CTS message defers its transmission until the ongoing transmission is complete. [4, 5].

A various range of BER within a given Virtual Connection characterizes wireless communication channels. Therefore, it is reasonable to create an appropriate probabilistic model of the wireless channel where time between failures has Generalized Erlang distribution.

Let us assume that a piece of information enters to the input of protocol module transmitter from the outside (for instance from upper level protocol) and then is transmitted over a given data channel (DC). This fragment must be recoded before it transfers. After conversion, received to the input information is transmitted over DC. After receiving acknowledgement from the subscriber about the successful reception of the fragment, output signal about transmission completion is generating by module-transmitter aside the receiver.

Unreliability of the transmission medium (DC) requires checking information about the proper transfer of information fragments, so that each block of information consists of checking information. n case of error detection, transmission is repeated until the correct block is received. In case of unsuccessful information transmission, data channel will be repaired. After DC recovery, information transmission is renewed starting from the damaged block.

III. Definition of Distribution Function of transmission of information with random length

The purpose of this paper is to determine Distribution Function (DF) of random length message transmission time Φ(t) in independent of quantity of blocks of message and quantity of repeated transmissions under the given characteristics of DC. We denote:

T- length of message;

n-quantity of blocks in message;

l-quantity of phases under Erlang distribution, i.e. the scheme of failures (errors) origin takes place according to which scheme process of failure can pass *l* phases (stages), before they really origin;

For analytical purposes of the above mentioned model we introduce: $\Phi_j^{(k)}(t,x)$ - the probability that fixed-length T message transmission (which consists of n blocks, each of them has a length τ_b) will be completed beginning with j-th block for a time less than t if: at the moment of t=0, DC was in k phase on failures (errors) and was transmitted x-th part $(x \in [0, \tau_b])$ of j-th block, where *l*-allowable quantity of block retransmissions, G (u) – DF of recovery time; α – a traffic rate of each phase distribution i.e. the duration of time intervals between successive moments of distortions origin have Erlang distribution A(u):

$$\mathrm{A}(\mathrm{u}) = \alpha\left(\alpha u^{l-1} e^{-\alpha u}\right) / (l-1)! \tag{1}$$

$\mathrm{F}(\mathrm{u}) = 1(t - \tau_{á})$ - DF of block length in common with control categories (where 1(t) – unit function, and $\tau_b = \mathrm{T/n}$ block length).

According to Definition:

$$\Phi_{\mathrm{j}}^{(\mathrm{k})}(t,x) = \begin{cases} \Phi_{j+1}^{(k)}(t,0), & at \quad \mathrm{x} = \tau_b; \\ 0, & \mathrm{at} \quad x > \tau_b \end{cases} \tag{2}$$

Where $\mathrm{k} = \overline{1, \mathrm{e}}; \quad j = \overline{1, n}.$ (3)

IV. Definition of Distribution Function of transmission of information with fixed length

Let us denote DF of probability with fixed-length by $\Phi(\mathrm{t,T})$ $(\mathrm{T} = \mathrm{n}\tau_{\mathrm{b}})$, and DF of messages with arbitrary (random) length, the following expression takes place:

$$\Phi(\mathrm{t}) = \int_0^\infty \Phi(\mathrm{t,u}) d\tilde{F}(u) \tag{4}$$

Where

$$\Phi(\mathrm{t,u}) = \sum_{\mathrm{k}=1}^{\mathrm{l}} \Phi_1^{(\mathrm{k})}(t,0)/l \tag{5}$$

denotes DF of transmission of messages with fixed-length u.

So given model is described by the system of integral equations:

$$\Phi_{\mathrm{j}}^{(\mathrm{k})}(t,x) = \int_0^t \frac{d_u F_j(x+u)}{1-F_j(x)} e^{-\alpha_k u} \Phi_{j+1}(t-u,o) +$$

$$+\int_0^{\mathrm{k}} \alpha_{\mathrm{k}} \frac{1-F_j(x+u)}{1-F_j(x)} e^{-\alpha_k u} \Phi_j^{(k+1)}(t-u, x+u) du; \tag{6}$$

$$\mathrm{k} = \overline{1, \mathrm{l}\text{-}1}; \quad \mathrm{j} = \overline{1, \mathrm{n}}. \tag{7}$$

$$\Phi_{\mathrm{j}}^{(\mathrm{l})}(t,x) = \int_0^t \frac{d_u F_j(x+u)}{1-F_j(x)} e^{-\alpha_l u} \Phi_{j+1}^{(l)}(t-u,0) +$$

$$+\int_0^{\mathrm{t}} (1\text{-}\mathrm{e}^{-\alpha_l u}) \frac{d_u F_j(x+u)}{1-F_j(x)} \int_0^{t-u} \Phi_j^{(1)}(t-u-\eta, 0) dG(\eta) \tag{8}$$

by denoting

$$\Psi_{\mathrm{j}}^{(\mathrm{k})}(t,x) = \left[1 - F_j(x)\right] \Phi_j^{(k)}(t,x) \tag{9}$$

equations (5) and (6) will take the form:

$$\Psi_{\mathrm{j}}^{(\mathrm{k})}(t,x) = \int_0^t d_u F_j(x+u) e^{-\alpha_k u} \Phi_{j+1}^{(k)} + \int_0^t \alpha_k e^{-\alpha_k u} \Psi_j^{(k+1)}(t-u, x+u) du; \tag{10}$$

$$k = \overline{1, l-1}; \quad \mathrm{j} = \overline{1, n} \tag{11}$$

$$\Psi_j^{(l)}(t,x) = \int_0^t d_u F_j(x+u) e^{-\alpha_l u} \Phi_{j+1}^{(l)}(t-u,o) +$$

$$+\int_0^t (1 - e^{-\alpha_l u}) d_u F_j(x+u) \int_0^{t-u} dG(\eta) \Phi_j^{(1)}(t-u-\eta, 0) \tag{12}$$

where $F(\tau_{á}^{+}) = 1; \quad F(\tau_{á}^{-}) = 0$

$$\Psi_j^{(k)}(t,0) = \Phi_j^{(k)}(t,0); \ \overline{F}(0) = 1; \ \ F(x) = 0, \quad x \in [0, \tau_{á}] \tag{13}$$

$$\mathrm{k} = \overline{1, l-1}; \qquad j = \overline{1, n} \tag{14}$$

Boundary conditions have the form:

$$\Psi^{(k)}_{n+1}(t,x) = \Phi^{(k)}_{n+1}(t,0) = 1; \qquad k = \overline{1, l-1}; \quad x \in [0, \tau_{á}] \tag{15}$$

Let us clarify equation (6):

first member in this equation denotes joint probability of that the transmission of j-th block (j=$\overline{1,n)}$ will be completed within the time equal to x: $d_u F(x+u)/(1-F_j(x));$ that within the time u, the transition to the next stage did not happen $\exp(-\alpha_l u);$ that within the time t-u, transmission of remained part of messages has been completed, starting from i+1-th block under e-th stage on failures (errors): $\Phi^{l}_{j+1}(t-u,0).$

The second summand of this equation – is joint probability of that the transmission of j-th block will be completed within the time x+u, under conditions that it was completed within the time x: $d_u F_j(x+u)/(1-F_j(x));$ probability of that the failure (error) occurs $(1-\alpha_1 \exp(-\alpha_l u))$ and DC is transferred for repairing: $dG(\eta);$ after repairing transmission is started from j-th block and is completed within the time: $\mathrm{t - u -} \eta$: $\Phi^{(1)}_{\mathrm{j}}(t-u-\eta,0).$

By using Laplace Transform in (6) and (8) equations, we'll obtain:

$$\Psi^{(\mathrm{k})}_{\mathrm{j}}(s,x) = e^{-(s+\alpha_k)(\tau_{\mathrm{b}}-x)} \Psi^{(k)}_{j+1}(s,0) + \alpha_k e^{(s+\alpha_k)x} \int\limits_{\mathrm{x}}^{\tau_{\mathrm{p}}} \mathrm{e}^{-\left(\mathrm{s}+\alpha_{\mathrm{k}}\right)\tau} \Psi^{(k+1)}_{j}(s,\tau)\, d\tau; \tag{16}$$

$$\Psi^{(\mathrm{l})}_{\mathrm{j}}(s,x) = e^{-(s+\alpha_k)(\tau_{\mathrm{p}}-x)} \Psi^{l}_{j+1}(s,o) + g(s)\Phi^{(1)}_{j}(s,0)\, \tilde{f}_j(s,x), \tag{17}$$

Where

$$\tilde{\mathrm{f}}_{\mathrm{j}}(s,x) = \int\limits_0^{\infty} e^{-su}(1-e^{-\alpha_e u})\, du F_j(x+u); \; j = \overline{1,n} \tag{18}$$

$$\tilde{\mathrm{f}}(\mathrm{s,x}) = e^{-s(\tau_{\mathrm{p}}-\mathrm{x})} - e^{-(s+\alpha_k)(\tau_b - x)}$$

$$\tilde{\Psi}^{(k)}_{j}(s,x) = \begin{cases} \Psi^{(k)}_{j}(s,x), & at \quad \mathrm{x} < \tau_b; \\ \Phi^{(k)}_{j}(s,0), & at \quad \mathrm{x} = 0; \\ \dfrac{1}{s}, & at \quad \mathrm{x} = \tau_b; \\ 0, & at \quad \mathrm{x} > \tau_b; \end{cases} \tag{19}$$

Boundary conditions have the following format:

$$\Psi^{(\mathrm{k})}_{\mathrm{j}}(t,x) = \Phi^{(k)}_{n+1}(t,0) = 1; \quad k = \overline{1,l}; x \in [0, \tau_b] \tag{20}$$

Moving to the differential in (2.4.5), we obtain:

$$\frac{\mathrm{d}\Psi^{(\mathrm{k})}_{\mathrm{j}}(s,x)}{dx} - (s+\alpha_k)\Psi^{(k)}_{j}(s,x) + \alpha_k \Psi^{(k+1)}_{j}(s,x) = 0; \quad k = \overline{1, l-1}. \tag{21}$$

$$\Psi^{(k)}_{j}(s,0) = \Phi^{(k)}_{j}(s,0) \tag{22}$$

$$\Psi^{(l)}_{j}(s,x) = e^{-(s+\alpha_k)(\tau_b - \mathrm{x})} \Phi^{(l)}_{j+1}(s,0) + g(s)\Phi^{(1)}_{j}(s,0)[e^{-s(\tau_{\mathrm{p}}-\mathrm{x})} - e^{-(s+\alpha_1)(s_b-\mathrm{x})}] \tag{23}$$

To simplify the calculations we introduce a new variable $\mathrm{y} = \tau_{á} - \mathrm{x}$ (where y - the time before finishing of block transmission). Equations (21) and (23) have the format:

$$\frac{d\Psi^{(k)}_{j}}{dy} + (s+\alpha_k)\Psi^{(k)}_{j}(s,y) - \alpha_k \Psi^{(k+1)}_{j}(s,y) = 0; \tag{24}$$

Where

$$\Psi^{(k)}_{j}(t,x) = \Psi^{(k)}_{j}(t, \tau_{á} - \mathrm{y}) = \tilde{\Psi}^{(\mathrm{k})}_{\mathrm{j}} = \Psi^{(k)}_{j}(s,y); \tag{25}$$

$$\Psi^{(k)}_{j}(t,0) = \Psi^{(k)}_{j}(t, \tau_{á}) = \Phi^{(\mathrm{k})}_{\mathrm{j+1}}(t,0); \tag{26}$$

$$\Psi^{(k)}_{j}(t,0) = \tilde{\Psi}^{(k)}_{j}(t, \tau_{á}); \tag{27}$$

$$\tilde{\Psi}^{(k)}_{j}(s,0) = \Psi^{(k)}_{j+1}(s, \tau_{á}) = \Phi^{(\mathrm{k})}_{\mathrm{j+1}}(s,0); \tag{28}$$

$$\tilde{\tilde{\Psi}}^{(l)}_{j}(s,y) = e^{-(S+\alpha_l)y} \Phi^{(1)}_{j+1}(s,0) + g(s)e^{-sy}(1-e^{-\alpha_l y})\Phi^{(1)}_{j}(s,0). \tag{29}$$

Using by the method of successive substitution: k = l, k = l-1, k = l-2, etc., we have:

$$\Psi_j^{(k)}(s,\omega) = \frac{1}{(\omega+s+\alpha)^{l-k}}\left[\frac{\alpha^{l-k-1}}{s} + \frac{\alpha^{l-k} g(s)\Phi_1^{(1)}(s,0)}{\omega+s}\right] \quad (30)$$

Taking the Laplace transform on argument y (corresponding operator ω), we obtain:

$$\tilde{\overline{\Psi}}_1^{(1)}(s,y) = \frac{1}{s}e^{-(s+\alpha)y} + g(s)\left[e^{-sy} - e^{-(s+\alpha)y}\right]\Phi_1^{(1)}(s,0). \quad (31)$$

Substituting into (31) y= τ_b and taking into account:

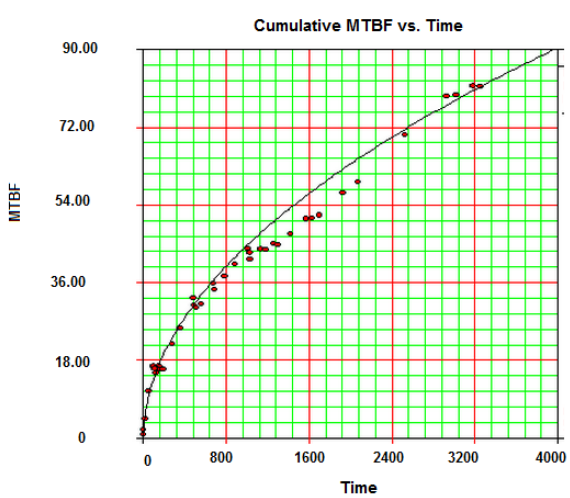
$$\tilde{\overline{\Psi}}_1^{(1)}(s,\tau_{\acute{a}}) = \overline{\Phi}_1^{(1)}(s,0) \quad (32)$$

The solution has the form:

$$\overline{\Phi}_1^{(1)}(s,0) = \frac{e^{-(s+\alpha)\tau_b}}{s\left\{1-g(s)\left[e^{-s\tau_b} - e^{-(s+\alpha)\tau_b}\right]\right\}} \quad (33)$$

Mean time Between Failures (MTBF):

$$T = -\left|s\overline{\Phi}_1^{(1)}(s,0)\right|'_{s=0} = -\tau_b + (\tau_b - \tau_{\hat{a}})\left[1-e^{-\alpha\tau_b}\right] \quad (34)$$



Basic scientific and practical results of this paper are as follows:

1) The analysis of factors influencing on the data channel, as on the complex technical system consisting of unreliable elements, was conducted
References
2) As an apparatus for investigation has been used: the apparatus of the theory of digital data transmission, reliability theory and queuing theory.
3) Distribution laws of random factors influencing the functioning of the data link were used.
4) Based on the analysis, distribution function of data transmission is selected as performance criteria of data transmission channel.
5) We consider a mathematical model of the data channel as a queuing system
6) Probabilistic-time characteristics with Erlang distribution of time of intervals between neighbouring failures were obtained in terms of the Laplace-Stieltjes

Conclusion

Numerical values of the efficiency and reliability of the data channel were obtained in the graph in a wide range of variation of the original data. The Approximate values of these parameters have been established. Conducted calculations have confirmed the feasibility of the theoretical results of this paper

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