Flipped Classroom Teaching in a Mathematics for Technology Course: Recommendations for Success

Jordan Allison

ABSTRACT

Mathematics is an important topic for computing and engineering students, but students often find the subject challenging, which emphasises the importance of using effective pedagogical strategies. The constructivist approach of the flipped classroom, where students learn new content out of class, and practice problems and engage in discussion in class, is raising in popularity. However, it is seldom reported how such an approach is used for teaching mathematics in the context of computing courses. This paper reports the experience and findings of implementing a flipped classroom approach for a foundation university module titled 'Mathematics for Technology'. A flipped classroom approach is shown to be successful in this context, but there are key considerations for its implementation. This includes factors such as constructive alignment, the desired learning and assessment outcomes, and the disposition of the student cohort. Recommendations are provided for practitioners who may be interested in implementing the flipped classroom approach in a similar context.

CCS CONCEPTS: Social and professional topics → Computing education; • Applied computing → *Collaborative learning*.

KEYWORDS: Flipped Classroom, Flipped Learning, Mathematics Education, Computing Education, Pedagogy

1 INTRODUCTION

Effective learning and student success is a key issue within higher education and particularly in computer science [21]. However, mathematics is a critical component for computer science and related fields such as engineering [1], but many undergraduate computing students struggle in this area [5], with mathematics often considered difficult due to its abstract concepts [23]. Consequently, mathematics teaching requires innovative pedagogical strategies [5], especially in the context of computing where students often have such a disparity of ability in the two subjects [8].

With computing and technology evolving rapidly [10], students will need to be able to adapt to these changes over time, not just during their university experience. Consequently, students must be taught or given the opportunity to learn for themselves and construct their own meaning to new developments. This approach has the theoretical underpinnings of constructivism, a long-standing learning theory originating with Piaget who described constructivism as the process where students construct their own systems of knowing which are unique to them as individuals [20].

Over time, other scholars such as Vygotsky and von Glaserfeld have created their own adaptions of constructivism; social constructivism, and radical constructivism respectively. The social constructivist theory of learning acknowledges that both social processes and individual sense making are important, including in the learning of mathematics [11]. This Vygotskian version of social constructivism emphasises the importance that social context plays in the learning of mathematics, where groups construct knowledge for one another and collaboratively create a set of shared artefacts and meaning. This social element is important to consider when applying and teaching mathematical concepts in the context of technology, since the subject itself is constantly evolving and changing, with practical applications that go beyond just the procedural knowledge of steps and algorithms that are commonly found in pure mathematics curricula. Hence, in a social constructivism view, students would construct knowledge from one another and their social interactions.

Meanwhile, radical constructivism focuses less on social elements and more so on the individual idiosyncratic creation of meaning [11]. It assumes that knowledge, however this is defined, is individual to the person based on their experience [26]. The key difference between the two views is that while Vygotsky emphasises how an abstraction of an experience may be compatible with the abstraction another has made, von Glaserfield argues that this can never be shown to truly be the same [26]. In the context of mathematics, this view is also important as learners naturally construct new knowledge on the foundations of their existing knowledge. For instance, how being able to differentiate an equation builds on the knowledge of manipulating basic algebraic expressions. Here students may be following a set of their own constructed rules, but as stated with radical constructivism, the knowledge individuals create do not tell us anything about reality, but only to help function in their environment [26].

Both views are important given the two types of subject matter in the course; pure mathematics, and the application of mathematics to technology, but regardless of which interpretation is taken, constructivism does not refer to any specific pedagogy [8]. However, what is important is the implications from taking this theoretical view of learning. Most notably is that learning is seen as an ongoing process where students create their own mental frameworks which are continuously refined over time [18], not a one-off occasion. This necessitates more of a student-centered approach to learning [1] with teachers helping students to construct their own knowledge, and so acting in more of a facilitator role [19]. One pedagogical approach of doing this is using the flipped classroom.

2 RELATED RESEARCH

The constructivist teaching approach of the flipped classroom is not a new concept, and has been used with positive results in computing courses [5, 14]. Different approaches and guidelines have been used when implementing a flipped classroom approach but overall, it is typically where theory is studied at home, with practical exercises or discussion taking place in the classroom [14]. There is extensive research of using the flipped classroom approach in mathematics education. For instance, some studies report how online resources being available for students out of class have supported the claim that online resources enhance mathematics learning [25]. Equally, one paper analysed the journal publications of mathematics flipped classroom studies where instructional videos were provided prior to face-to-face class sessions [16]. The authors found through a synthesis of 61 studies that the flipped classroom approach benefited student learning through greater instructor feedback, peer-assisted learning and interaction, and a greater amount of class time to apply concepts. Hence, a flipped classroom approach allows students to learn theory at their own speed and allows a greater amount of class time to be used to address issues of student understanding or exploring concepts in more depth.

A flipped classroom approach contrasts to one of the most popular taxonomies of learning; Bloom's taxonomy. Widely used in higher education for course design [15], Bloom's taxonomy is a framework that classifies educational learning objectives into different levels of complexity or specificity. It is one of the most widely used taxonomies in stating learning goals for computing [17], and ranges from lower order thinking skills to higher order thinking skills. Correspondingly, the six main elements are remembering, understanding, applying, analysing, evaluating, and creating. However, some work has explored whether Bloom's taxonomy is applicable for computer science [8], with Johnson and Fuller [15] questioning its appropriateness and hypothesising that in computing teaching, the aim is what they term 'higher application'. A flipped classroom approach flips the 'normal' classroom so these 'higher application' skills can be the focus in class, as opposed to leaving students to go away and develop these skills on their own. Consequently, the flipped classroom approach flips Bloom's taxonomy so that the higher order thinking activities (i.e. analysing, evaluating, and creating) take place in the classroom, with the lower order thinking activities taking place outside of scheduled class time [5]. Hence, the effectiveness and implementation of the flipped classroom is interesting to consider in a range of computing topics, and one such area which has typically been neglected is how a flipped classroom approach is used within mathematics modules as part of a computing course [5].

3 CONTEXT OF PRACTICE

This paper reports on the experience of implementing a flipped classroom teaching approach for a foundation university module titled 'Mathematics for Technology'. This was a new module for the academic year 2020-2021 and served as a preparatory module before students begin their respective programs of study. Twenty-two students attended this module and consisted of a variety of computing and cyber security students, engineering students, and games design/programming students. Consequently, both the teaching and learning approaches, and the subject topics needed to be broad enough to cater for the variety of student needs and interests.

This module was delivered during the midst of the COVID-19 pandemic, which imposed emergency remote teaching across education worldwide leading to many educational institutions closing and shifting some or all teaching online [9]. This resulted in the module delivery consisting of a two-and-a-half-hour online session each week, and a one-hour face-to-face session every other week. Therefore, given that online teaching is generally perceived as lower quality than face-to-face [6, 9], with effective learning and student success more difficult to achieve due to the lack of humanity in the experience [22], this only emphasised the importance of a structured teaching approach.

To guide the teaching approach, the concept of 'constructive alignment' [3] was heavily applied which focuses on creating a teaching and learning environment where the learning outcomes, teaching content, and assessment are all intrinsically linked together [3]. As per the course validation documentation, a variety of topics were expected to be included in this module such as calculus and linear algebra. Meanwhile, there was an emphasis on ensuring that students could apply these concepts to their respective courses. For example, moving beyond adding or multiplying matrices to applying matrices in the context of cryptography such as in a hill cipher. Hence, learning outcomes focused on students gaining the ability to solve problems regarding different mathematical topics but also for students to understand how these topics are applied within computing and engineering, and this needed to be reflected in the assessment.

Typical assessments in mathematics curricula are often timebased exams which focus on being able to solve for an answer with little implication for understanding the application of mathematics beyond the exam. However, teachers should convey to students what mathematics is all about as opposed to having students just learn a set of rules and procedures [1]. Besides, practice work (such as solving mathematical problems) is only one part of the learning process, and is primarily applicable to procedural knowledge [12], whereas writing assignments can provide students with opportunities to demonstrate their conceptual understanding, and allows a broader range of students to participate effectively [12]. Given that each student has different prior experiences and perceptions [13], and the students on this module would advance to different subject areas, it was deemed that an assessment was required which allowed students to have the opportunity to reflect and focus on their own topics of interest.

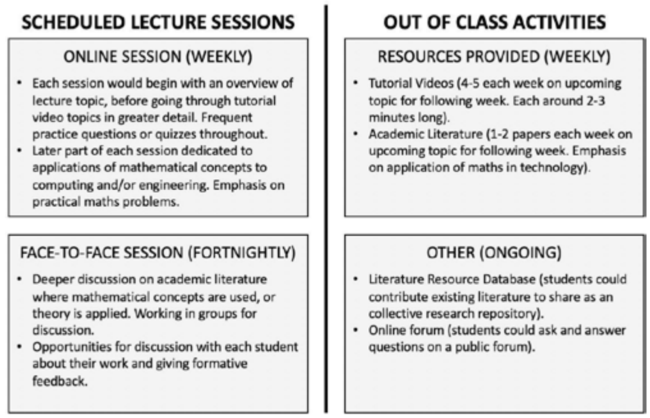

Figure 1: Flipped classroom approach used in teaching 'Mathematics for Technology'.

The assessment created was a written assessment where students needed to explain the purpose of mathematics in the context of computing and engineering, by referring to three example applications of their choosing, where each used a different mathematical concept. Students were required to incorporate academic literature, in addition to showing the calculations or code used to explain the applications they had chosen. Students were also required to comment on and explain the steps of their calculations to demonstrate their understanding of the process. By structuring the assignment in this way, it would help students scaffold their learning by forcing them to organise their ideas and thoughts on mathematical concepts to applications [12]. This approach was mostly successful, with student assessments covering a wide range of applications which including the following:

- Resistor-capacitor parallel circuits
- Games design (e.g., 3D modelling, transformations, calculus in physics engines)
- Ultrasound imaging (sinusoidal waves)
- Aerospace engineering
- Cryptography (matrices)
- Satellite systems and communication
- Construction (e.g., using integration for finding length of cable required)

4 THE FLIPPED CLASSROOM APPROACH

Based on the context of practice, there were four considerations to balance in developing the teaching approach; the weekly online sessions, the fortnightly face-to-face sessions, resources to be provided weekly for students out of class, and other ongoing considerations or resources. The plan and activities for these four considerations are outlined in Figure 1.

The key to a flipped classroom approach is the link between the out-of-class and in-class activities [13, 16]. Each week students would be provided with access to tutorial videos explaining key topics and mathematical concepts that would be the focus of the following weeks lecture. The tutorial videos were created by the

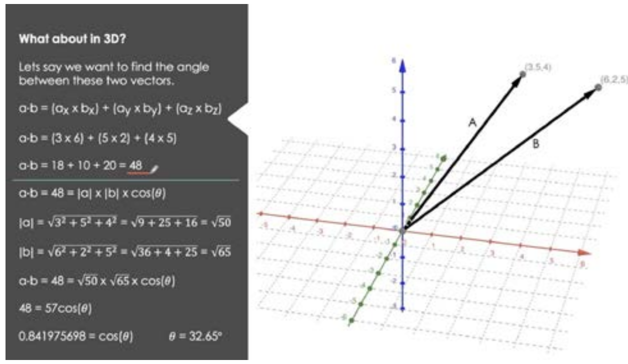

Figure 2: Using Geogebra as a visual aid in 3D coordinate systems.

author and followed the recommendations by Choe et al. [6], who discusses student satisfaction and learning outcomes in asynchronous online lecture videos. For each week, around four of five tutorial videos would be created, and each were short videos of around 2-3 minutes long that focused on quick, sharp introductions and explanations to individual topics. Topics were either explained on a whiteboard or through tools such as Geogebra due to the benefits that diagrams and illustrations provide [23]. This was particularly important when explaining topics such as vectors in 3D coordinate systems (Figure 2), as some students may better understand concepts visually, as opposed to only seeing calculations. Students could view the tutorial videos at any time to support their learning, benefitting those who may wish to take their learning at their own pace and what works for them with the tutorial videos receiving excellent student feedback. The other resource provided to students weekly as part of the flipped classroom approach was one or two academic papers relating to mathematical concepts applied to computing and engineering. These papers would serve as 'starter' papers for later discussion in class.

The weekly online sessions began with an overview of a lecture topic, before proceeding to go over the topics covered in the tutorial videos. The online lecture slides covered a greater depth than the tutorial videos, but the tutorial videos (providing they were watched by students), would mean that there was a greater amount of class time to explore concepts further. Towards the end of each lecture there was a focus on practical mathematical problems related to computing and engineering, with the provided academic literature serving as the starter point for discussion. During each session, interactive elements and practice questions for students were included with different approaches used throughout the module delivery. Sometimes an online quiz format was used, other times a worksheet was created for students to enter answers into, or sometimes students were asked questions where they were required to answer either verbally or by entering their answer into the chat box functionality of Microsoft Teams. Regardless of which method was used, feedback would be provided, and correct answers explained. However, the key problem in this online format was that students could easily dis-engage from the process, by turning off their camera, and not answering any questions, and merely being passive observers of the online session.

Face-to-face communication is preferred when content goes beyond the simple dissemination of information [19], and therefore, the face-to-face sessions were used for the focused discussion of mathematical applications and answering student queries. Before each face-to-face session, students were asked to research applications relevant to that week's topic to bring to the lecture for discussion. These could then be used for interactive engagement, peer instruction [2] or classroom discussion which would make the time more useful for the students [5], especially as this would heavily link to the assessment requirements. Furthermore, structuring the sessions in this way allowed the author to provide individual formative feedback more effectively, and avoided the demoralising situation of having to 'spoon-feed' young adults [7]. Importantly, this approach also allowed the author to gauge whether students understood mathematical applications as opposed to stand-alone calculation solving. However, these sessions were challenging, as some students could not attend due to self-isolation, while some students did not conduct any research at all in some instances. Unfortunately, a flipped classroom approach does have the tendency to be challenging for both students and educators [7]. Students may not be familiar with this teaching approach [16], while some students may dislike it as they cannot passively receive information [2].

There were two other resources and considerations that were out of class activities and ongoing throughout the module delivery. On the virtual learning environment, a forum was created where students could ask and answer questions, both to the module tutor and each other. However, this was rarely used, with only three students regularly using it to ask questions to the module tutor. Furthermore, a literature resource database was created where students could upload academic papers and share them with the rest of the class. To avoid the situation of students putting in no effort to receive this information, they were incentivized by having to upload at least two papers themselves to view what else was added by other students. Unfortunately, this initiative was also unsuccessful with only one student adding papers to the database. On reflection, this initiative was probably not communicated effectively to students, and given that students were provided with one or two papers each week, they may not have viewed this additional activity as beneficial for them. Nevertheless, the fact that some students contributed to these activities can be seen as a positive. Therefore, these initiatives will be continued for the academic year 2021-2022 as they have clear potential for success. However, clearer expectations and guidance will be provided to students in the very first session regarding these initiatives to encourage greater student engagement.

5 CONCLUSION AND RECOMMENDATIONS FOR PRACTITIONERS

A key problem faced by computing lecturers is bridging the gap between learning outcomes, what to teach, and how to assess what students have learned [24]. This paper has discussed the approach taken to address this issue and discussed the implementation of a flipped classroom approach in the context of a 'Mathematics for Technology' module delivered during the COVID-19 pandemic. Unlike some authors who used a flipped classroom approach in some topics and not others, and then compared student assessment results [5], it is more difficult to quantify the effectiveness of the flipped classroom approach for individual mathematical concepts in the context of this study due to the nature of assessment. However, the focus of the module was on developing a student's readiness for later university study, to consider academic literature, and apply meaning to mathematical concepts, not just the ability to solve them. Formative feedback was given to students regarding their ability to solve calculations, but a single synoptic assessment was used to assess how well students understood where mathematical concepts could be applied.

It was clear how some students gained a clear understanding of mathematical concepts and their applications by exploring topics in great depth. This was the intended outcome, and it was hoped that deep learning of the topics would be achieved by the students where they could apply their learning to applications comprehensively [4]. On the other hand, the assessments of weaker students tended to be more generalized, and only covering the foundational aspects of mathematical topics. This is more akin to surface learning, where students may have had a greater focus on getting the assignment out the way and meeting course requirements as opposed to doing it well [4]. Nevertheless, given the circumstances, and the overall quality of student assessment and student feedback, the teaching approach used can be considered a success. However, as the same approach was used throughout the module, and the flipped classroom approach was not used in isolation for only specific topics, it cannot be identified whether students would have fared better if a flipped classroom approach was not used. Nevertheless, implementing a flipped classroom approach resulted in some key lessons being learnt during the process, which can serve as recommendations or guidelines for practitioners willing to adopt a similar approach.

5.1 Recommendations

The following recommendations have been identified from the combination of existing literature and the teaching experience reported:

- Set clear expectations for flipped learning from the very start of each module and clarify expectations as sessions progress.
- Ensure there is a clear link between in-class and out-of-class activities.
- Create clear short tutorial videos for individual topics. These should be created with sufficient lighting with high quality audio and video.
- If possible, create tutorial videos that have embedded questions within them as another avenue for students to assess learning.
- Incorporate quizzes weekly to assess student knowledge and to incentivize students to complete the out of class activities.
- All elements of the teaching and learning should be focused on constructive alignment. There should be clear links between learning outcomes, content, and assessment.
- Encourage students to engage in group work and collaborative discussion regarding mathematical applications.
- Obtain feedback on your teaching practice to gain another perspective on how to improve.
- Students vary in background, experience, and willingness to engage. Therefore, early in the module delivery, build a rapport with students and experiment with different ways to increase student engagement.
- Obtain consistent weekly student feedback (not just at the end of module delivery), to understand the needs of students more effectively so teaching methods can be tailored appropriately.

This paper reported the experience of implementing a flipped classroom teaching approach for a foundation university module titled 'Mathematics for Technology'. It is clear how a flipped classroom approach can be successful but there are many considerations if choosing to implement such an approach. This paper addresses the limited amount of research which considers how this approach is used in the context of teaching mathematics for computing students, while also providing a perspective of how such an approach can be used in a blended environment that consisted of both online and face-to-face sessions. The author acknowledges how this is a work in progress, and the implementation of the flipped classroom is not a singular process. The results of this paper and reflecting on the module delivery will inform the future design and teaching of the module for the academic year 2021-2022 and beyond. Through taking slightly different tactics in teaching this module over time, it will be possible to gain a clearer understanding of what is most and least effective in implementing a flipped classroom approach for university mathematics teaching in the context of computing and engineering.

REFERENCES

[1] Michal Armoni. 2011. Looking at Secondary Teacher Preparation Through the Lens of Computer Science. *ACM Transactions on Computing Education* 11, 4 (2011), 1–38.https://doi.org/10.1145/2048931.2048934

[2] D Berrett. 2012. How 'flipping' the classroom can improve the traditional lecture. *The Chronicle of Higher Education* 12, 19 (2012), 1–14.

[3] John Biggs. 2003. Aligning teaching for constructing learning. *Higher Education Academy* 1, 4 (2003).

[4] John Biggs and Catherine Tang. 2011. *Teaching For Quality Learning At University* (4 ed.). Vol. 2011. Open University Press, Maidenhead.

[5] Michael Bradford, Cristina Muntean, and Pramod Pathak. 2014. An analysis of flip-classroom pedagogy in first year undergraduate mathematics for computing. In *IEEE Proceedings - Frontiers in Education Conference, FIE*. IEEE, 1–5. https://doi.org/10.1109/FIE.2014.7044072

[6] Ronny C. Choe, Zorica Scuric, Ethan Eshkol, Sean Cruser, Ava Arndt, Robert Cox, Shannon P. Toma, Casey Shapiro, Marc Levis-Fitzgerald, Greg Barnes, and Rachelle H. Crosbie. 2019. Student satisfaction and learning outcomes in asynchronous online lecture videos. *CBE Life Sciences Education* 18, 4 (2019), 1–14. https://doi.org/10.1187/cbe.18-08-0171

[7] Brian Robert Cook and Andrea Babon. 2017. Active learning through online quizzes: better learning and less (busy) work. *Journal of Geography in Higher Education* 41, 1 (2017), 24–38. https://doi.org/10.1080/03098265.2016.1185772

[8] Tom Crick. 2017. *Computing Education: An Overview of Research in the Field*. Technical Report. Royal Society, London. 1–38 pages.

[9] Tom Crick, Cathryn Knight, Richard Watermeyer, and Janet Goodall. 2020. The Impact of COVID-19 and "Emergency Remote Teaching" on the UK Computer Science Education Community. In *United Kingdom & Ireland Computing Education Research conference.* ACM, Glasgow, 31–37. https://doi.org/10.1145/3416465. 3416472

[10] Stuart W. Elliott. 2017. *Computers and the Future of Skill Demand*. OECD Publishing, Paris. https://doi.org/10.1787/9789264284395-en

[11] Paul Ernest. 1994. What is social constructivism in the psychology of mathematics education?. In *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education*, Vol. 2. Lisbon, Portugal, 304–311.

[12] Kris H. Green. 2002. Creating successful calculus writing assignments. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies* 12, 2 (2002), 97–121.https://doi.org/10.1080/10511970208984021

[13] Mick Healey, Abbi Flint, and Kathy Harrington. 2014. *Engagement through partnership: students as partners in learning and teaching in higher education*. Higher Education Academy, York.

[14] Antti Herala, Erno Vanhala, Antti Knutas, and Jouni Ikonen. 2015. Teaching programming with flipped classroom method: a study from two programming courses. In *Proceedings of the 15th Koli Calling Conference on Computing Education Research*. ACM, New York, NY, USA, 165–166. https://doi.org/10.1145/2828959. 2828983

[15] Colin G. Johnson and Ursula Fuller. 2006. Is Bloom's taxonomy appropriate for computer science?. In *Proceedings of the 6th Baltic Sea conference on Computing education research Koli Calling 2006*. ACM Press, New York, NY, USA, 120–123. https://doi.org/10.1145/1315803.1315825

[16] Chung Kwan Lo, Khe Foon Hew, and Gaowei Chen. 2017. Toward a set of design principles for mathematics flipped classrooms: A synthesis of research in mathematics education. *Educational Research Review* 22 (2017), 50–73. https://doi.org/10.1016/j.edurev.2017.08.002

[17] Susana Masapanta-Carrión and J. Ángel Velázquez-Iturbide. 2018. A Systematic Review of the Use of Bloom's Taxonomy in Computer Science Education. In *Proceedings of the 49th ACM Technical Symposium on Computer Science Education - SIGCSE'18*. ACM, New York, NY, USA, 441–446. https://doi.org/10.1145/3159450. 3159491

[18] Richard E. Mayer. 2018. Thirty years of research on online learning. *Applied Cognitive Psychology* 33, 2 (2018), 152–159. https://doi.org/10.1002/acp.3482

[19] Manuela Paechter and Brigitte Maier. 2010. Online or face-to-face? Students' experiences and preferences in e-learning. *Internet and Higher Education* 13, 4 (2010), 292–297.https://doi.org/10.1016/j.iheduc.2010.09.004

[20] Jean Piaget. 1950. *The Psychology of Intelligence* (1 ed.). Routledge, London.

[21] Tom Prickett, Julie Walters, Longzhi Yang, Morgan Harvey, and Tom Crick. 2020. Resilience and Effective Learning in First-Year Undergraduate Computer Science. In *Proceedings of the 2020 ACM Conference on Innovation and Technology in Computer Science Education*. ACM, Norway, 19–25. https://doi.org/10.1145/ 3341525.3387372

[22] Theresa Redmond and John Henson. 2020. The transcendent power of remix : Cultivating creativity, story, and student voice in online learning. In *Teaching, Technology, and Teacher Education During the COVID-19 Pandemic: Stories from the Field.* Association for the Advancement of Computing in Education (AACE), 77–84. https://www.learntechlib.org/p/216903/

[23] Zerrin Ayvaz Reis and Sebnem Ozdemir. 2010. Using Geogebra as an information technology tool: Parabola teaching. *Procedia - Social and Behavioral Sciences* 9 (2010), 565–572.https://doi.org/10.1016/j.sbspro.2010.12.198

[24] Christopher W. Starr, Bill Manaris, and RoxAnn H. Stalvey. 2008. Bloom's taxonomy revisited. In *Proceedings of the 39th SIGCSE technical symposium on Computer science education - SIGCSE '08*. ACM Press, New York, New York, USA, 261–265. https://doi.org/10.1145/1352135.1352227

[25] Evangelia Triantafyllou and Olga Timcenko. 2015. Student perceptions on learning with online resources in a flipped mathematics classroom. In *CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education*. Prague, Czech Republic, 2573–2579.

[26] Ernst von Glasersfeld. 1995. *Radical constructivism: a way of knowing and learning*. The Falmer Press, London. 210 pages.