# PRIMARY TRAINEE TEACHERS' CHOICE OF MATHEMATICAL EXAMPLES FOR LEARNING AND THE RELATIONSHIP WITH MATHEMATICAL SUBJECT KNOWLEDGE

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## Abstract

When teachers plan to teach mathematics, they draw on many examples to either demonstrate a concept or provide opportunities for learners to practise skills and procedures. The examples used by primary trainee teachers, it is suggested, are often chosen without suitable consideration of learners' strengths, weaknesses or misconceptions. Whilst there has been research on the choice of examples by teachers in secondary mathematics, detailed empirical research of primary mathematics or for trainee teachers is relatively scarce. In this study, two cohorts of final year trainee primary teachers were invited to submit lesson plans for analysis and a sample group was interviewed to try to identify the theoretical frameworks trainees use for planning mathematics and their approaches to choosing examples for learning. The data collected was then analysed using a multiple case study approach against a conceptual framework based on the Knowledge Quartet research of Rowland *et al.* (2009) and the development of the notion of example spaces by Watson and Mason (2005). The analysis sought to identify commonalities in the way the group of trainees approached planning mathematics and draw insights on their rationales for choosing mathematical examples. Each trainee's planning was scrutinized against the theoretical background in the literature and conclusions were drawn regarding the methods of planning adopted, the examples chosen and the possible links between these actions and the trainees' levels of mathematical subject knowledge. Evidence from the study appears to show that trainees do not make use of theoretical frameworks when planning mathematics lessons, examples are chosen from existing sources such as textbooks and websites, and any modifications are made with differentiation as a key factor rather than mathematics pedagogy, with trainees' subject knowledge playing a minimal role in the planning process.

I declare that the work in this thesis was carried out in accordance with the regulations of the University of Gloucestershire and is original except where indicated by specific reference in the text. No part of the thesis has been submitted as part of any other academic award. The thesis has not been presented to any other education institution in the United Kingdom or overseas.

Any views expressed in the thesis are those of the author and in no way represent those of the University.

Signed ... ........... Date ....17/5/2010...........

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# Contents

**List of Tables** **Page**

**List of Appendices** **Page**

# Chapter 1: Introduction

## 1.1 Rationale

This study has its roots in personal experience going back over a number of years and is based on experience firstly as a classroom practitioner and more recently as an academic. During a period as a primary teacher with a mathematics degree background, I was always interested in how mathematics was taught and learnt by primary age pupils. In my early days as a teacher, there was no national curriculum and planning for mathematics was through localised or school-based schemes of work. The examples used in teaching were very largely taken from textbooks of the time, such as the 'Alpha' and 'Beta' series (Goddard *et al.*, 1979) or graded examples for each year group such as 'The Four Rules of Number' (Hesse, 1982). In those early days of my practice, I was aware that as a means of saving time, I would occasionally construct a set of examples of my own which I wrote on the blackboard, with chalk, for the children to work through. The thought that went into such examples, on reflection, involved identifying what I considered to be a 'simple' example, showing sufficient features of the concept I wanted children to learn, but being within their capability to carry out and solve in whatever format was required. For children who could quickly work out the set of examples provided, I then had to add further examples to the list to challenge them and develop their learning. This often meant using larger numbers or setting a problem within a context where the calculation needed was less obvious without some prior interpretation. The connecting factor amongst the examples I used was that they each had to fall under the heading of the concept I was trying to teach and in completing them, the children would meet a prescribed objective.

At that point in my teaching career, I had little or no awareness that examples might be categorised in any form of typology and I knew very little about research in mathematics education or any theoretical frameworks that might describe approaches to teaching and learning mathematics. Much of my thinking was based on my own enthusiasm for mathematics and I recall working with children aged 9-11 years on topics such as the Möbius strip, simple combinatorial problems, chessboard problems, games such as 'Nim' and number patterns beyond multiplication tables including Pascal's triangle, prime numbers and powers of numbers.

Through taking posts of responsibility for mathematics in different primary schools, I was able to focus on how to improve attainment in mathematics through improving its teaching and learning, which implicitly included the careful selection of examples. By this stage, the National Curriculum (DfEE, 1999) had been implemented in England and Wales and my mathematics teaching was 'reined in' to cover in greater depth the curriculum content as prescribed. The introduction of statutory tests for 7 and 11 year old pupils in 1994 seemed to further restrict my teaching, with national guidance and local management insisting on coverage of 'testable items'. Moving onto have overall responsibility for pupil attainment in a school as a headteacher gave me a different outlook on how mathematical attainment was being reached through approaches to teaching and learning, since I was able to observe the different teaching styles of the teaching staff, and subsequent learning by pupils.

By this stage I had also begun to be involved in mathematics organisations such as the Mathematical Association and the Association of Teachers of Mathematics and encountered a number or research papers which broadened my horizons to consider mathematics education in a wider context. This influenced my thinking in doing a Masters degree, focussing on the mathematical beliefs of trainee primary teachers as well as developing my skills as a researcher.

After moving into higher education as a senior lecturer in primary mathematics education, my interest turned more specifically towards how trainee teachers approach the teaching and learning of primary mathematics, and I became interested in how I needed to take account of their various mathematical backgrounds and attainments as I looked to develop them into good teachers of primary mathematics. There were some students whose only mathematical attainment was a pass at GCSE level, (General Certificate of Secondary Education, usually taken at age 16 in England) whilst some had progressed to GCE 'A' level, (Advanced Level General Certificate of Education, usually taken at age 18 in England) or even worked in business or financial institutions where mathematical applications of one form or another was part of their everyday work. Not only were their levels of mathematical competence varied, so were their levels of interest and enthusiasm in mathematics as a subject or body of knowledge, skills and understanding. This meant that the overall mathematical experiential backgrounds were broad in their nature, and it was interesting to see how this might impinge on their developing role as trainee teachers and how this might affect their choice of examples.

Also during the early part of my career as an academic, I began to attend seminars, conferences and colloquia, and it was at these that my awareness of mathematics education research developed to a greater extent. In particular, attendance at a Mathematics Colloquia at Homerton College, Cambridge in the Michaelmas Term of 2002 inspired my interest in mathematical examples when Anne Watson discussed the notion of 'example spaces'. I was able to attend the opening plenary lecture of the annual conference of the Mathematics Education Research Group of Australasia at Deakin University, Geelong in June 2003 when Watson had developed her ideas further and together with awareness of the development of the Knowledge Quartet by Tim Rowland, Peter Huckstep and Ann Thwaites at Cambridge, the seeds of this research study were sown.

The research seeks to examine primary trainee teachers' choices of mathematical examples within their teaching, set against current debates around mathematical subject knowledge. This arose initially from my early experiences as a teacher and from awareness during teaching sessions and from observing trainees teaching mathematics on school placements that primary trainees seemed to lack depth of mathematical subject knowledge and were unable to select examples for their lessons based on sound pedagogical reasons.

For the purpose of this study, I considered only the primary phase, since that is my phase of speciality and experience, although there may be parallels with secondary trainees that could constitute a separate study. I moved from being a primary practitioner to develop an academic career after moving from classroom teaching, through school management and into university lecturing and research.

Such a range of experience offers a good base to draw from work and experience in both areas of work, and utilise it effectively in helping achieve the research aim of developing a framework to help teachers decide how to choose examples effectively in the primary classroom. However, being in a position to draw on practitioner and academic practice may cause problems in terms of defending a particular approach or framework due to a possible overlap of ideas and therefore a lack of clear justification for any method which tends towards one area of practice rather than the other. Such a dilemma may be resolved as the research progresses if the outcomes lead away from a combined position, either closer to the academic end or towards the practitioner end. Whatever the outcome, there remains commitment to making the research purposeful and useful for teachers to draw on in future mathematics teaching and learning.

Whilst working on this study, I was fortunate to be part of a Nuffield series of meetings for Mathematical Knowledge in Teaching, during which I was able to discuss my ideas with colleagues from the mathematics education community which proved to be an invaluable source of help in terms of developing my depth of knowledge about the field and expanding my capabilities as a researcher.

## 1.2 Structure of the thesis

The research presented here will be set out in four following chapters. Chapter 2 will be a consideration of recent and relevant literature pertaining to the study of mathematical examples. It will draw primarily on the development of the Knowledge Quartet (Rowland *et al.*, 2009) and work on examples and example spaces (Watson and Mason, 2005), using the typologies within each of those aspects of research to analyse the data from this study. The literature will be examined to identify appropriate intellectual tools with which to analyse the data collected.

Chapter 3 will then set out the methodological aspects of the research, clarifying how the research design was established, the features of the methods being used and ethical considerations. The choice of case study as a design suitable for examining primary teacher trainees' approaches to choosing mathematical examples will be argued, including deliberation of the positive and negative aspects of case studies for research. Data collection methods are then examined, with reflection on my role as a data collector and the selection procedures for the cases are discussed in line with the aims of the study. Finally, an analytical procedure is set out that will assist the research of teaching and learning in mathematics focusing on trainees' choice of examples.

In chapter 4, the data is analysed, discussing the findings and describing them using evidence from interview transcripts and lesson plans. The identities of the seven case study trainees will be described, along with an analysis of their data, examined from the perspective of the literature and its key themes and the research questions listed at the end of chapter 2. Qualitative data analysis is then used to develop the findings which emerged from the data and these are supported with evidence from interviews and lesson plans, selected from two cohorts of trainees at one United Kingdom university. Lesson plan evidence is taken from those lessons that focused on aspects of teaching and learning number, since this attainment target was the most common amongst the collected set of plans.

Finally, chapter 5 will outline the main findings from the study in relation to the research questions and consider implications of the research both for my own practice and for recommendations for those involved in initial teacher training. In this chapter the original aims of the study are revisited, namely to examine pedagogic considerations when primary teacher trainees choose mathematical examples and how the trainees' mathematical subject knowledge impacts on their choice. This will lead to a review of the research questions that were intended to be answered by the research. The main findings from the research will be reviewed and summarized and the extent to which the research questions were answered will be addressed. The research outcomes will then be discussed against existing research and both the chapter and thesis will end with some implications for my own practice as a mathematics educator and some key recommendations that could be presented to those working in the field to consider ways of improving practice in the area of trainees choosing mathematical examples.

# Chapter 2 Review of the Literature

## 2.1 Introduction

The learning of mathematics and its pedagogy are key elements in the training of primary school teachers and approaches to teaching mathematics to trainees are continually being developed in the field of mathematics education. This research seeks to develop a more critical approach to supporting classroom practice of trainees by developing effective ways of using mathematical subject knowledge in choosing and planning the use of learning examples. A number of writers have addressed the issue of choosing examples in the secondary mathematics field (Zaslavsky and Peled, 1996; Watson and Mason, 2005; Bills and Bills, 2005) and deduced some key principles regarding teaching and learning through appropriate choice of examples.

In the field of primary mathematics education, recent studies have focussed on the development of teachers' subject knowledge (Shulman, 1986; Goulding *et al.*, 2002; Huckstep *et al.*, 2002; Rowland *et al.*, 2009) and it is this area in particular that will be explored to see how it impacts on primary trainee teachers' choice of examples. In this chapter literature associated with mathematical subject knowledge will be reviewed, exploring firstly the forms of knowledge identified within mathematics and secondly mathematics education will be considered, exploring the role of higher education in providing a mathematical knowledge base for primary teachers through the course structure.

This review will also offer a consideration of how mathematical subject knowledge can influence and enhance the choice of mathematical examples used by teachers in their lessons, and the role higher education might play in ensuring that the relationship between subject knowledge and choice of examples can be usefully developed.

## 2.2 Mathematical Knowledge for Teaching

### 2.2.1 Knowledge for Teaching

Huckstep *et al.* (2002) considered the issue of mathematical knowledge by drawing on an ancient Greek example (*The Meno,* Plato, 1956 edition) which demonstrates how Socrates deals with a young boy's misconception over the relationship between the side length and area of a square. In the encounter, Socrates engages onlookers to observe the elicitation of mathematical misunderstanding from the young slave. He then traps the boy into assuming the relationship between the dimensions holds before apparently leading him out of it through the use of some carefully chosen questions. To do this successfully, Socrates' calls upon his mathematical knowledge. He also makes use of his ability to correct the boy's responses through more questioning, which requires a particular level of knowledge to do this. As a result of this exchange, the boy comes to learn the correct relationship, but this depends on Socrates' knowledge of the misconception and his ability to introduce a degree of cognitive conflict.

The knowledge base which Socrates calls upon includes and extends beyond mathematics knowledge by itself which demonstrates an important aspect of the mathematical subject knowledge which teachers require in order to be at their most effective in enabling learners to develop their own mathematical understanding. A teacher's role is to help the learner achieve understanding of the subject matter. However, in order to do so, the teachers themselves need to have good knowledge of the subject matter, and this is a necessary but not sufficient condition. A teacher with deep, connected mathematical knowledge for teaching is more capable of helping his or her students achieve a meaningful understanding of the subject matter. Not only is the knowledge of mathematics essential, but the deeper ability to question and prompt learners into understanding it. The scope of what mathematical knowledge consists of will now be explored in more detail.

Vinner (1983) described 'concept image' as a central notion, being a set of all pictures that have been associated with that concept in a person's mind, together with the set of properties associated with that concept. If teachers have a good match between a concept and their understanding of it, then their teaching is more likely to be effective and this research sought to examine the extent to which trainee primary teachers could match concepts and understanding through examining their lesson planning, drawing to some extent on Vinner's notions of subject knowledge and concept image. In forming their concept image, teachers will have been exposed to a range of examples which they can distinguish between more precisely as their concept image approaches that of the underlying concept. This then forms the basis of the teacher's understanding from which they draw their examples for teaching and defines their epistemological position.

A teacher's concept image would also allow for the connections between different aspects of mathematics to be drawn from when there are similarities in the thinking processes needed for particular concepts, a kind of generated knowledge which comes into being for the teacher when required. Depending on the learning situation a teacher faces, there will be choices available to them for their examples which are the best type for the learner's current conceptions. The notion of connected knowledge is one which is explored later in relation to work by Askew *et al.* (1997) and the Knowledge Quartet (Rowland *et al.*, 2009), where there is reasonable agreement in the work of various authors on this idea. The ideas presented in this section demonstrate aspects of the field which have come under scrutiny in recent times, and form one of a number of strands of thinking against which the choice of examples of trainee primary teachers can be analysed.

### 2.2.2 Categories of Teacher Knowledge

Teachers' knowledge for classroom practice was conceptualised in a seminal work by Shulman (1986) in which he sets out categories of teacher knowledge. His work has been used widely in developing approaches to developing and assessing teacher knowledge, particularly in work arising from the implementation of Circulars 10/97 and 10/98 (DfEE, 1997, 1998). In these documents, curricular coverage for Initial Teacher Training (ITT) in the United Kingdom was set out. Three of Shulman's seven categories focus directly on what can be called content knowledge, these being subject matter knowledge, pedagogical content knowledge and curricular knowledge.

Shulman (1986) also describes how subject matter knowledge (SMK) will be different in different areas but uses the notions of substantive and syntactic knowledge. The former includes key facts, concepts and principles in a discipline whilst the latter is more about the nature of enquiry in a given field, and how new knowledge is introduced and accepted in the community of that discipline. Ball (1990) echoes Shulman's constructs of substantive and syntactic knowledge in any discipline by making a distinction between knowledge *of* mathematics (meanings underlying procedures) and knowledge *about* mathematics (what makes something true or reasonable in mathematics). Ball found in studying trainee teachers that, for example, teachers had significant difficulties with their understanding of the *meaning* of division by fractions. Most could do the calculations, but their explanations were rule-bound, with a reliance on memorising rather than conceptual understanding.

A similar study by Ng (1998) showed that meaning could be given, for example, to fractional division through the use of illustrative diagrams to complement the teaching of division purely by algorithm. Such an approach could be useful to primary trainee teachers whose mathematics knowledge is limited to algorithmic approaches in their personal use and their teaching, and in this study it may become apparent that trainee teachers use diagrams to support their subject knowledge.

Ma (1999) compared United States (US) and Chinese teachers' knowledge of 'fundamental' mathematics and found that whilst the US teachers had a mostly limited and sometimes faulty understanding, the Chinese teachers were able to do the calculation, provide examples of the concept in context and discuss concepts underpinning the understanding of division. Ma concluded that an ignorance of particular mathematical concepts will not be addressed by increasing pedagogical knowledge, an issue which could have significant implications in teacher education, where a focus on pedagogy has become prevalent in recent years. Pedagogical content knowledge (PCK) is for Shulman:

> The ways of representing the subject which makes it comprehensible to others...[it] also includes an understanding of what makes the learning of specific topics easy or difficult.
>
> (1986, p.9)

PCK is difficult to describe and define, but is essentially about conceptualising the link between knowing something for oneself and enabling others to know it. Brophy (1991) noted that pedagogical subject knowledge (equivalent to Shulman's PCK) is influenced by teachers' beliefs about mathematics, and with associated values and attitudes towards what is involved in teaching mathematics. One broad but clarifying definition of pedagogical subject knowledge, which helps to set the context in which the trainee teachers for this research are working, is given by Aubrey (1994) who says:

> Pedagogical subject knowledge includes knowing what knowledge, concepts and strategies children bring to learning, their misconceptions as well as their understandings, and the stages through which they pass towards mastery of topics within a subject area.' (p.106)

In choosing their examples, it is supposed that trainee primary teachers draw on a combination of their subject knowledge and pedagogical knowledge, and this section has outlined these types of knowledge, taken from key literature which helped to define the types of knowledge in a way which the field of mathematics education has embraced and endorsed in developing approaches to improve trainees' knowledge for teaching mathematics.

### 2.2.3 Auditing Subject Knowledge and Trainees' Approaches

The Office for Standards in Education (OFSTED) in England produces annual reports which focus on specific curriculum areas. The main findings of their reports for the years 2003/04 and 2004/05 were reported on by Abbott (2006), who describes how the strengths being found in primary school mathematics are often characterised by teaching that targets pupils' understanding and a good balance of the requirements of curriculum coverage and depth of learning. Further, where this balance is good, a key feature is the level of teachers' subject knowledge. The best teachers were found to understand key mathematical concepts and how best to teach them, and it was encouraging that the government's own defenders of educational standards were reporting on mathematical subject knowledge as being a key feature in the best teaching and learning. From the literature, it seems there is a strong argument for a connection between SMK and the effective teaching of mathematics. This relationship involves a combination of both cognitive and affective dimensions.

One dilemma, however, is whether the requirements for ITT in terms of auditing subject knowledge to improve classroom practice is also creating more anxiety and dislike of mathematics for non-specialist mathematics teachers, or whether it acts as a useful lever for development. It would not be helpful to make trainees jump through more hoops simply to increase instrumental understanding. Further research into the process of auditing and remediating subject knowledge will give an insight into how provision could best be improved, and this would have an implication for teachers in the way they are then informed about making suitable choices of examples for the classroom.

Further debate should also focus on determining exactly what the content of the mathematics SMK should be, and thus it is crucial that ITT and continuing professional development (CPD) play a role in strengthening teachers' SMK, since it is this knowledge which will be drawn on in their choice of examples. This research will seek to ascertain how trainees regard mathematical subject knowledge and their development of knowledge through their course.

The role of knowledge in the development of a primary trainee teacher has been examined by mathematics educators and government alike, each arguing for the content of a teacher education curriculum in mathematics. The concern with subject knowledge, particularly for primary trainees, has been growing since Alexander, Rose and Woodhead (1992) felt that strengthening teachers' subject knowledge would contribute to the aim of improving standards in mathematics. A later evaluation of the first year of the National Numeracy Strategy (Ofsted, 2000) in England located weaknesses in teachers' mathematical subject knowledge.

The numeracy strand of the Primary National Strategy, introduced in the UK in 2006, was the dominant non-statutory guidance for primary schools in terms of mathematics teaching, and was underpinned by the statutory National Curriculum, implemented in 1988 and revised in 2000. In this review, the original National Numeracy Strategy (NNS) (DfEE, 1998) provides a backdrop to trainees' teaching, having been the backbone of primary mathematics for over ten years and remains a significant aspect of their university mathematics programme in terms of planning lessons and choosing examples.

The '*Framework for teaching mathematics from Reception to Year 6*' (DfEE, 1999) set out the required teaching content for mathematics, as well as how to plan and teach groups and how to assess progress against the learning objectives. For many teachers the NNS Framework offered a structured, progressive programme to support planning, and gave examples of the numeracy skills children should acquire each year. For teachers who are competent in teaching mathematics, the NNS seemed an unnecessary addition to a series of government interventions in education, but since most primary teachers are not mathematics specialists, many may have seen the document as support to cover up their insecurities about mathematical subject knowledge. This would have enabled them to become formulaic deliverers of mathematics lessons.

The NNS content in terms of its method prescription generates concerns about the guidance and how teachers use it, so there is a need for research into teachers' levels of subject and pedagogical content knowledge, and how this transfers into choices of learning examples for children.

This is being considered in this research, but it is proposed that there is evidence that the guidance for schools is based not on research-based evidence relating to teaching and learning approaches, knowledge of subject or pedagogy, but that the Numeracy Strategy was brought about as much by political whim as by concerns derived by worthy educational motives, and in the intervening years, the Strategy has been the subject of critique from educationalists and mathematics education researchers alike, some of whose work (Askew *et al.,* 1997; Thompson, 2003; Haylock, 2006) will be considered here.

There are a number of aspects of the NNS to which writers have taken exception, for example, the Strategy's sequence for moving from mental to written calculation is described as flawed, but can offer appropriate tactics for teachers to ensure the objectives can be met in a pedagogically sound way (Thompson, 2003). Another key writer is Askew, who studied teacher effectiveness in numeracy (Askew *et al.,* 1997). Askew contributed to studies focussing on teaching styles and how they impact on mathematical learning, and whilst the 1997 study was completed before the introduction of the Numeracy Strategy, the findings implied a strong need for sound subject knowledge and a connectionist approach (Askew *et al.*, 1997) to teaching and learning, something not evident in the examples which then appeared when the NNS was implemented just two years later, and which has already been highlighted in an earlier section (2.2.1).

Haylock (2006) has worked on identifying those aspects of subject knowledge which are crucial to effective teaching, but which are not directly related to a particular policy or strategy. In other words, he deliberately considered subject content and pedagogical content knowledge which is inherent in what is understand about mathematics teaching and learning, an approach to be commended, and which has certainly helped in educating new teachers of primary mathematics.

Each of these writers has considered how best to develop a cognitive approach to teaching and learning mathematics. However, in contrast to this approach, the NNS seemed to encourage teachers to adopt an approach which restricted learners' deep understanding of the facts, skills and concepts associated with mathematics.

One way that teachers' subject knowledge can be developed and improved is through effective CPD, and many local authorities provide training days which can help teachers address their weaknesses, whether these are in content knowledge, pedagogical knowledge or in confidence as a result of having not taught a topic for a long time at a particular level. Such training could provide teachers with opportunities to engage in learning new mathematics, and could involve the creation of powerful learning situations with colleagues, followed by reflection on the processes that took place during the learning. Even in this situation, appropriate choice of content becomes a critical factor in enabling teachers' learning to develop effectively.

Following publication of the Williams Review of Primary Mathematics (2008), a key recommendation for England was that a mathematics specialist should be deployed in every primary school, and a programme of professional development was to be established to train these specialists. The debate in higher education institutions during the period of bidding to deliver the programme was rightly concerned with issues of content and quality, as well as reaching an agreed definition of deep subject knowledge, a phrase used by Williams in describing the criteria for specialists. Any such definition could perhaps have been focussed around the issues considered by Zaslavsky (1995).

Zaslavsky describes two important issues regarding content for teachers' CPD which can be compared with the choice of learning examples for pupils. A topic would need to be selected that is suitably demanding in order to require the participants to genuinely engage in new learning. The implication here is that teachers would need to feel comfortable learning some mathematics which would be extending their PCK. Also, selecting familiar topics from the curriculum may not create real learning experiences for teachers who already know and teach these topics well. For pupils it can be argued that the balance to be struck is between presenting learning situations which are suitably demanding and challenging, and those which are already within the pupil's comfort zone and thus inspire neither challenge nor learning.

Audits and consequent remediation were considered to be a strategy for addressing the problem of trainees' weak mathematical subject knowledge. There was considerable concern about trainees' anxiety and how best to address the weaknesses without damaging trainees' self confidence (Barber and Heal, 2003). Murphy (2003) found a tension between trainees seeing the audit process as useful or just a hoop to jump through on the way to qualification. Specific difficulties with geometry and probability were examined by Mooney *et al.* (2003) and similarly for reasoning and proof by Goulding *et al.* (2002), and in each case the researchers found a need for improved subject knowledge by teachers in order to become more effective at enabling learners to understand the concepts.

The complex nature of categorising teachers' subject knowledge is nothing compared to trying to explore its use in action. There are immediately more complicated inter-relationships between variables, such as the level of teacher knowledge and pupil progress. Aubrey (1997) carried out a comprehensive review of literature on teaching competence, comparing US expert and novice teachers. Her conclusions argue that there is a central importance of disciplinary knowledge to good primary teaching. Aubrey's (1997) model, which subsumes SMK within pedagogical subject knowledge, seems to be a useful one since it acknowledges the importance of teaching in the whole enterprise. This view is supported by Hodgen (2003) who refers to the fact that teachers use mathematical knowledge not so much for doing mathematics but rather for teaching it. Nonetheless, attention to SMK would seem to be necessary within any consideration of pedagogy.

The literature referred to in this section has been used to set out the main issues with regard to identifying misconceptions and weaknesses in trainees' mathematical knowledge and ways of supporting trainees with developing their knowledge across the range of mathematical topics. Issues around the link between mathematical subject knowledge and the choice of examples that teachers might make when engaging learners with mathematical concepts will now be explored, so that the connections between these two areas can be established to some extent before examining the research data.

One of the key ideas from literature is that the examples used by teachers or trainees will be influenced significantly by their mathematical subject knowledge. If this is weak or has significant gaps, then the examples are not likely to be chosen specifically for the concepts being worked on, but rather will be randomly selected on the basis that the example is within the right topic and therefore must be useful.

Morris (2001) worked with Postgraduate Certificate in Education (PGCE) trainees to ascertain the level of mathematical subject knowledge they could demonstrate in test conditions and then ways of addressing the gaps. She discovered that many trainees regard themselves as weak mathematicians but do not have sufficient confidence to do anything about it, or they see themselves as confident in the subject even if they scored poorly in the test, still regarding themselves as being capable of teaching what they regard as elementary content. Further analysis of her data led to a categorisation of three types of trainee.

Firstly the 'nettle-grasper' who took charge of their learning when they realised they had deficiencies in their knowledge, and whose confidence ensured they could boost their knowledge through appropriate study. Secondly the 'ostriches' who literally buried their heads in the sand over their problems with content knowledge, and finally the 'mananas' who realised they had a problem but never quite managed to do anything about it. Morris concluded from her research that mathematical knowledge levels are best addressed in future on teacher education courses through the compilation by trainees of a portfolio to demonstrate their knowledge and any work they have done to alleviate misconceptions.

A tool used by researchers in Melbourne, Australia to identify misconceptions through a diagnostic means is the Teacher Education Mathematics Test (TEMT) which supports assessment of trainees towards teaching the Victorian 'Curriculum and Standards Framework' (Board of Studies, 1995; 2000). Ryan and McCrae (2005) analysed the test results to produce a diagnostic mapping which they refer to as a 'Kidmap'. From this Kidmap, decisions can be made by trainees and their tutors about the types of questions they found difficult and the priorities they should have for addressing their subject knowledge gaps. Whilst this in itself is helpful, it is the discussions within tutor groups and teaching sessions which the authors identified as significant in supporting the development of subject knowledge, since in these situations the examples generated enable the trainees to enhance their understanding of the concepts as well as develop useful examples for their own teaching, which also enhance and improve not only the trainees' general mathematical subject knowledge, but the particular knowledge form discussed previously, namely pedagogical content knowledge (PCK).

## 2.3 Examples in mathematics teaching

This study examines the examples chosen by a number of final year Bachelor of Education primary trainee teachers. The types of examples will be analysed during consideration of the lesson plans, but it is helpful to have a typography and an understanding of the possible types of example that may be encountered. In order to analyse trainee teachers' examples, it is necessary to have a clear categorisation, or framework, against which to relate the trainees' choices. For this purpose, one such categorisation is that presented by Watson and Mason (2005). Whilst they focus on the role of learner-generated examples and the advantages of using such examples to enhance mathematical learning, they set out a number of definitions of examples which help to clarify the differences between types of examples that might be constructed and used by teachers or their pupils. Using the notion that mathematics is about making general statements regarding the 'actions carried out on objects' (Mason, 2010), the categorisation of examples which Mason and Watson set out uses the assumptions that all examples are used to enable a learner to generalise from them. They present the following types:

- Illustrations of concepts and principles
- Placeholders instead of general definitions and theorems
- Questions worked through in textbooks or by teachers (worked examples)
- Questions to be worked on by students (exercises)
- Representatives of classes
- Specific contextual situations

These types can be used in the process of exemplification, that is:

> to describe any situation in which something specific is offered to represent a general class with which the learner is to become familiar – a particular case of a generality (Watson and Mason, 2005, p.4).

In the current study, the types of example will relate to mathematical concepts at the primary level, and so examples of the example types listed above might be as follows:

- To illustrate concepts and principles, use two fractions that show equivalence, such as 2/4 and 3/6.
- A placeholder instead of a definition might be rearranging the corners of a triangle by tearing and re-aligning to show the angle sum is 180°.
- Questions worked through by teachers or in textbooks will be demonstrated in chapter 4 when the results of the research are analysed and discussed.
- Questions for students to work through will also appear in chapter 4.
- Class representatives could be identifying a sequence of digital roots of the multiples of 8 and then looking for patterns within the sequence.
- Specific contextual situations might include a problem solving situation that draws on particular knowledge or skills to solve it.

Research on teachers' choice of examples is scarce according to Bills *et al.* (2006) although some work has been carried out that either relates to this feature of teaching mathematics, or leads to the possibility of further research using frameworks that have been constructed from the limited research on this precise topic. For this purpose, the work of Watson and Mason (2005) and Rowland *et al.* (2009) are considered.

Examination of teachers' choice of mathematical examples for this study was partly inspired by the writings of Marton, who first began to discuss learning through variation in experiences back in the 1980s. In subsequent work he discussed learning in terms of a process of discerning variation between similar items presented almost simultaneously.

The ways in which learning experiences can be varied are described by Marton *et al.* (2004) as the 'dimensions of variation', and by using this approach a teacher could select structured examples which provide variations for the learner to notice and subsequently learn from. Marton *et al.* (2004) describe how, from a learner's perspective, what is actually learned in a lesson, what they call the 'lived object of learning', comes about primarily through the teacher's use of variation. Following on from the point raised earlier, learners can only learn an 'object' when it is presented in comparison to something with which it differs. They give the example of colour-blind learners being unable to take up the opportunity to learn the colours red and green because they see no difference between them. Recalling my undergraduate mathematics experiences, not being able to carry out Laplacian transformations was due to the inability to identify the variation between different examples.

The variations discussed here, as determined by the teacher for the learner's benefit, create what Marton calls a 'space of learning', which refers to what it is possible to learn in a given situation. Such a space is generally created and developed through use of language, and this theory of learning is therefore perhaps following in the tradition of Jerome Bruner.

This view of learning has been taken and developed by Watson and Mason (1998) exploring in depth what it means to use dimensions of variation in mathematical examples, for the benefit of classroom teachers. In this, they have considered how a series of examples could be given to learners in which a sequence of increasing constraints is used to focus the learning on a particular concept.

For example, 'think of 2 numbers', 'think of 2 numbers which add to 12', and 'think of 2 numbers that add to 12 and multiply to 27' would offer a range of starting points, but quickly narrow down the possibilities to a specific solution. In later research, Watson and Mason (2002) consider the use of examples to help learners explore mathematical statements by considering the truth of statements as being always true, sometimes true and never true. In this way they conclude tentatively that powerful exercises that engage learners with mathematical structure can be designed by controlling the dimensions of variation through considering dimensions of possible change, the ranges of change and the range of permissible variation.

The use of examples opens up the possibility that tasks and activities for learners can be explored in terms of the language of affordances. Greeno argues that the way individuals process information in an example needs to shift to an understanding of the information that is available to them in the example. He describes affordances as 'qualities of systems that can support interactions and therefore present possible interactions for an individual to participate in' (Greeno, 1998, p.9).

Within the system of an example-based mathematics lesson, there exist constraints which force the learner to adopt particular concepts from a given example of set of examples, and how an individual adapts to those constraints demonstrates the attunements the learner makes, in other words the regular ways in which an individual learner participates in social practices.

The collection of constraints and affordances generate what Greeno termed 'ecology of participation', and the examples presented in a classroom offer the learners a range of participation possibilities. Since the examples within a given lesson may differ only slightly, the affordances and constraints need to be described very specifically, and the language of variation (Marton *et al.*, 2004) is a helpful model for this.

Atkinson *et al.* (2000) reported that learning from examples is not a new phenomenon, and that from the mid-1950s through to the end of the 1970s there was considerable research by cognitive and educational psychologists into the paradigm of learning by example (Bruner *et al.*, 1986). In these studies, students' abilities to identify a particular member of a target concept (for example, a right angled isosceles triangle) were measured. This was after they had been exposed to numerous instances and non-instances of it. The idea behind this approach is that through examining a range of possible examples, they were able to ascertain the key features of the target concept (namely that the triangle must have a right angle and two equal length sides meeting at the right angle).

Bills *et al.* (2006) carried out documentary analysis of texts and other writings dating back to the 15$^{th}$ century in order to ascertain the extent to which examples have played a significant role in the teaching and learning of mathematics. They considered the types of example that has existed over the years, and categorised them as being 'worked solutions', 'diagrams, symbols and reasoning', 'concepts versus procedural applications', and 'raw material for generalising'.

They further argue for the use of examples as a means of presenting a range of possibilities from a particular class of mathematical objects, from which the learner can appreciate the 'range of permissible variation' within that class, an idea which uses many of the ideas put forward by Watson and Mason (2005).

Based on the researcher's knowledge and experience from teaching, examples can be simply regarded as falling in one of two categories (Rowland, 2008). Examples can be seen as inductive objects from which the learner is given an example *of* something. This 'something' is usually general in nature, for example the notion of static angle, or the fact that multiples of 5 end in 0 or 5. Such examples use particular instances of a generality, and this use of such examples is commonly found in educational practice to represent abstract concepts.

An example of this might be the use of 43 – 27 in demonstrating column subtraction. In such an example, the digits 4, 3, 2 and 7 are carefully selected from a choice of options to ensure the appropriate learning. The range of choices of digits open to the teacher in this example is described by Watson and Mason (2005) as the 'range of permissible variation', meaning that in this instance, the possible variations that can be used are limited to the range of the digits 0 – 9. For pedagogic reasons to do with requiring children to understand how the process of decomposition of tens into units is carried out there are further restrictions on the range of digits available in each position of the calculation.

For mathematical concepts, examples can make the abstraction process simpler for the learner. Examples unified by concept formation for the learner allow subsequent ones to be assimilated, even from outside learners' experience, described by Skemp (1979) as reflective extrapolation. However, findings such as those of Dahlberg and Housman (1997) should be noted, who reported that:

> A student's initial understanding of a concept often persists despite the presence of examples and information which conflict with this initial understanding. (p.283)

Looking now at a second category of examples, these might be described as practice-oriented, and often referred to as 'exercises'. They can be formed from a set of all possible examples within a particular concept, for example, when preparing an exercise on multiplying a 2-digit number by a single digit, there will be only a limited number of possibilities and a subset of these will form the exercise. Such an exercise needs to be carefully designed to avoid becoming a mundane task, something that for Watson and Mason means that 'attempts to answer closed questions are checked out against teacher's answers' (2006, p.94).

Exercises are designed to provide practice for learners to enable them to become fluent and accurate in procedures, however, poorly designed exercises can be completed with little engagement with the intended concept and no improvement in fluency due to the difference from one example to the next. Examples need to be selected to be suitably challenging and motivating for learners and yet provide for the 'desired construction of understanding' (Muir, 2007, p.518).

There is a third pedagogical deployment of examples, but these fit into the former inductive category and is far less common in practice. These would be the presentation of arguments through particular cases for explanation, and this type was evident amongst a group of practising teachers in research by Bills and Bills. They found that instead of using a 'generic example to think about first examples, rather, they are seeing examples as a means to build up an argument' (2005, p.151). The construction of these examples requires considerable pedagogical skill. Whichever category of examples are used, there is a responsibility on the teacher to undergo a reflective process of informed choice to achieve the most effective outcomes for the learners.

In the discussion so far, examples have been considered as illustrative selections from a class of similar situations or as unsolved problems on which learners can operate to seek solutions. A further category of examples might be called 'worked examples' in which a problem or situation is presented along with a typical solution which is being used to model for the learner how their solutions might be generated. Chi *et al.* (1989) considered the use of worked examples particularly in mathematics, physics and computer programming.

The role of an example and it is presented for understanding is now considered. Examples have always been used by teachers to play a role in explanation. If an explanation is given in very general terms, an example obtained by specification of variables may be used as a reason from which the statement is derived by induction. Thus an example may become a basis for understanding the general statement and is perhaps a pedagogical adage that 'we learn by examples.'

Educators think of paradigmatic examples, or instances that can best explain a rule or method, or a concept. As a learner, one also looks for paradigmatic examples whilst learning something new. The problem with this, however, is that before having a grasp of the whole domain of knowledge being learned, the learner is unable to distinguish a paradigmatic example from a non-paradigmatic one. This inevitably leads to mistakes being made, both wrong choices and wrong generalisations, since generalisation comes from examples. Also, as the example is normally represented in some medium (enactive, iconic, or symbolic), the features of the representation may be mistaken for the features of the notion thus exemplified.

Michener (1978) used four types of examples, these being:

- Start-up examples – for using as the basis for definitions or early conjectures
- Reference examples – widely applicable cases against which to test concepts
- Model examples – generic cases
- Counterexamples – to help clarify the extent of a concept

These types are defined more by the teacher's intentions rather than the learner's ability to distinguish between them, and it is acknowledged by Watson and Mason (2005) that examples in one of Michener's categories could serve equally well in a different category, certainly for examples in the first three cases above.

Counterexamples are very specific examples because they falsify conjectures. If a learner reasoned that since 7 is odd, all multiples of 7 will be odd, 2 x 7 = 14 is a counterexample. The role of the counterexample is important in defining a concept, but creating counterexamples can cause difficulties.

For instance, Zaslavsky and Ron (1998) found that novice teachers regarded counterexamples as exceptions to a rule, but that the rule still held. Zaslavsky and Peled (1996) reported on a study which focussed on how experienced teachers and trainee teachers of mathematics generated counterexamples, in their case in the teaching of binary operations where they were looking for an example of a binary operation which had the commutative property but not the associative property. Zaslavsky and Peled tried firstly to identify the difficulties faced by teachers when generating examples in a particular topic and secondly to reveal possible sources of difficulty. This latter category was presumed to include some or all of the following: incomplete subject matter knowledge, inability to process any existing knowledge, misconceptions around that body of knowledge and its associated conceptual structures and insufficient logical knowledge. In their study, the inhibiting factors were assumed, and subsequently found, to be directly related to the level of subject matter knowledge and pedagogical content knowledge which became so crucial in later studies of primary trainees by Rowland *et al.* (2003, 2009). Zaslavsky and Peled found that experienced teachers had an advantage over trainees in that despite not always making significant use of advanced subject matter knowledge, they were, however, able to be more productive in their generation of counterexamples.

They thus produced fewer incorrect counterexamples in what was to them a more familiar body of knowledge and skills, or 'content-space', as Zaslavsky and Peled called it. In further exploring this outcome, they discovered that in the course of teaching, the experienced teachers develop a deeper conceptual understanding of the topic and so are more capable of working flexibly within it.

However, for both groups of teachers, the act of teaching, which requires a continual provision of examples, often on the spur of the moment in a lesson, seems to affect their willingness to take risks and their fluency in producing appropriate examples.

Finally in this section a mention for non-examples. These are not the same as counterexamples but have a role in demonstrating different cases within a given concept. For instance, 15 is a non-example of the set of even numbers. This leads to the notion that a collection of non-examples for a particular concept could be examples in their own right of a different concept, so 15 is an example of the set of odd numbers.

Examples as categorised earlier by Watson and Mason (2005) do not occur in isolation. They tend to be part of collections of examples that are each particular cases of something general, and these collections of examples are described as example spaces. Within a given example space, the individual examples will have some commonalities whereas there will always be minor differences between the examples. These differences, or variations, help define generality by expressing the range of features within the definition of the generality. In considering the boundaries of an example space, other ways of categorising examples become helpful in terms of their role and function for the learner. Since 'most people's examples are modified versions of examples they have encountered elsewhere' (Watson and Mason, 2005, p.62), many examples encountered in primary school demonstrate what Schwarz and Hershkowitz (2001) describe as prototype ambiguity.

This means that the way examples are represented encourages learners to focus on particular aspects of an example which could unduly bias their understanding. One such case is that of the square, which if presented with its sides not parallel to the edge of the page is often regarded by learners as a 'diamond', since they relate it to the name of something encountered in stories, for instance.

In considering the examples chosen by the research sample trainee teachers, it is possible that the range of examples will not be as extensive as the list given above, since the way trainees tend to teach is restricted by their own knowledge level and the limitations imposed by the statutory curriculum. As Watson and Mason record; 'most teachers and curricula are bound by a fragmentary knowledge-based approach' (2005, pp.24-5). This means that an example offered by a trainee teacher is already shaped by that trainee's understanding and intended use of the example in a particular situation. If the learner is then unaware of the role of the example in the learning context in which they find it, they may not see it as representative of the concept they are learning or be able to deduce generality from it. Further, learners will only make sense from those examples available to them, either from the teacher or another source. They can be misled if all the examples they see for a given concept are so similar that the generality is hidden. For example, primary pupils may invariably see triangles on a horizontal base, hexagons that are regular, or work out fractions by cutting up pictures of cakes or pizzas. When teachers of mathematics generate learning examples in their lessons, the range of suitable choices can be influenced by the earlier mentioned inhibiting factors.

Other recent research which has a bearing on the issue of choice of examples is that of Bills and Bills (2005). They interviewed trainee teachers and their school mentors about the choice of examples made during mathematics lessons and discovered that the examples which trainees began their lessons with were often quite demanding conceptually and did not allow the learners to grasp the concept easily. Their conclusion asks both trainees and mentors to consider the first 'simple' examples, and keep them simple for a while afterwards to allow pupils to see enough examples to begin to grasp the concept. Many examples they saw would not satisfy the requirement of a generic example or paradigm, or stand as 'particular to illustrate the general' (p.46). Bills and Bills suggest that trainees are often given advice to use examples which give pupils a straightforward procedure, but which do not extend their thinking or prepare them for a range of variation that may be met subsequently, whether in real life contexts or the examination room.

One way to consider the difficulty of examples, given the findings of Bills and Bills, is to express 'ease' or 'difficulty' in terms of the range of change of aspects of the examples being used. For example, when working with numerical problems, moving from smaller numbers to larger ones, extending from natural numbers to integers and then to rational numbers will give the teacher the opportunity to vary the examples to suit the learning needs of different pupils. Alternatively, by considering the dimensions of variation within a given situation, a teacher might choose to vary the representation being used in order to provide a different interpretation, or change an aspect of the examples not previously changed, such as number of digits, position of decimal point and so on.

This section has considered the variety of types of examples that might be used by teachers, from the perspective of knowing about what types are available. However, there is now a need to consider how use of these various types is manifested within classroom practice, and this is achieved in the following sections which draw on the theoretical framework of Rowland *et al.* (2009) which they termed the Knowledge Quartet.

## 2.4 The Knowledge Quartet

In the late 1990s when UK teacher training institutions were required to audit trainees' subject knowledge, the 'Subject Knowledge in Mathematics' (SKIMA) group was established to explore the relationship between SMK and teaching performance in school, as judged by observing tutors. Huckstep *et al.* (2003) found that effective classroom teaching of elementary mathematics was associated with secure SMK. Their research focused on the way trainees' mathematics content knowledge could be observed to be 'played out' in practical teaching during school placements.

In order to determine a possible link between subject knowledge and classroom performance, Huckstep *et al.* (2003) tried to answer a key research question which was 'in what ways is (novice elementary) teachers' mathematical knowledge made visible in their teaching?' (Rowland, 2008, p.276) Since the intention was to identify how mathematical knowledge was evident in teaching, the research was designed as a classroom observation study.

However, to set this up in a way which would give representation from both upper and lower primary classes with trainees who were identified across a range of attainments, they went about answering this research question by giving an audit of mathematics knowledge to a group of primary trainees following a one-year PGCE course. The scores on the audit were used to categorise the trainees as 'high', 'medium' or 'low'. A selection of 12 trainees from each group was then observed teaching mathematics on two separate occasions with the lessons being observed and videoed for later analysis. The researchers then wrote a 'descriptive synopsis' of each lesson, using the collected data firstly for coding and then to form 18 categories which identified aspects of the trainees' mathematics teaching. The categories were slimmed down through a process of rationalisation and negotiation into four dimensions which, after a period of writing 'analytical accounts' of each lesson and from discussion and feedback from a conference presentation (Huckstep *et al.,* 2002), form the Knowledge Quartet, a framework which became the focus for working with primary trainee teachers to support the development of their mathematics teaching.

This framework, developed using a grounded theory approach (Glaser and Strauss, 1987), has parallels with work by Ball and Bass (2003). However, whereas Ball and Bass developed ideas around the notions of SMK and PCK, separating out the earlier categories described by Shulman (1986), Rowland and the SKIMA group from Cambridge chose to focus on the 'classification of the situations in which mathematical knowledge for teaching surface in the classroom' (Rowland, 2008).

The four dimensions of the Knowledge Quartet, named as 'Foundation', 'Transformation', 'Connection' and 'Contingency' will now be described in the following sections.

### 2.4.1 The Foundation Dimension

Aptly named, this first of the four dimensions is set against trainees' theoretical background and beliefs. It focuses on their learning and preparation to be teachers of mathematics and makes no link to their actual classroom practice. However, it provides the theoretical underpinning form which the other three dimensions follow. The type of knowledge described by the Foundation dimension is that which is learned during trainees' personal and pre-service education, that is, 'in the academy,' (Rowland, 2008, p.288), the period before they take up classroom teaching roles in employment. In that regard, it is a dimension which links appropriately to this study which seeks to examine the choice of mathematical examples by pre-service trainee teachers. Rowland *et al.* (2005, p.260) regard such knowledge as having the 'potential to inform pedagogical choices and strategies in a fundamental way', with 'fundamental' being a rational and reasoned approach based on knowledge beyond imitation or habit to allow decisions to be made. The Foundation dimension is made up from three key components, these being knowledge and understanding of mathematics itself, knowledge of mathematics pedagogy and beliefs about mathematics and its learning. The first of these components relates to the work described earlier by Ma (1999) in criticising the use of procedural and instrumental understanding for even the most elementary topics in mathematics.

It also includes what Rowland *et al.* (2005) describe as the use of mathematical vocabulary, not as being correct, but as being both careful and deliberate. The pedagogical aspect includes awareness of research in mathematics education, which includes in recent years, for example, that which moved elementary teaching in number from partition and place value teaching to more holistic teaching of number concepts, including the shift, as reported by Thompson (1997) from pencil and paper algorithms to flexible mental methods. The last aspect of this dimension, beliefs, covers beliefs about mathematics and the philosophical positions which define the nature of knowledge in mathematics, beliefs about the purpose of mathematics education and beliefs about the best conditions in which to learn mathematics.

Each aspect of the Foundation dimension will be part of the trainees' experiences in this study and so an awareness of trainees' positions in terms of their Foundation knowledge will be useful in helping to describe their rationale for their teaching approach and subsequent choices of examples. The extent of trainees' Foundation knowledge will be ascertained through considering the original codes which combined to form the Foundation dimension, for example, the extent to which trainees adhere to using textbooks for examples, their awareness of the purpose of the topic and its methods, their overt subject knowledge, the extent to which they concentrate on procedures and their own use of mathematical terminology.

### 2.4.2 The Transformation Dimension

The second dimension is that of Transformation, the first of those which focuses on classroom practice, including both planning and teaching of mathematics. It is named in connection with the way Shulman (1986) discussed how a teacher can have capacity to transform their own mathematical content knowledge into a form that learners can access due to the pedagogic power of those forms. These forms can include representations of the knowledge into analogies, illustrations, explanations, demonstrations and, critically for this study, examples. Each of these forms suggest that the teacher is behaving in a way which acts towards one or more pupils 'and which follows from deliberation and judgement informed by foundation knowledge' (Rowland *et al.*,2005, p.262). Huckstep *et al.* (2003) acknowledge that their work does not attempt to make a direct link between mathematical knowledge and competence in teaching, but they introduce in the transformation dimension the notion of the appropriateness of trainees' choices of examples as well as identifying some of the ways in which examples are poorly selected.

One of the key elements of the Transformation dimension is that of 'representation', which Rowland *et al.* (2009) suggest is ways of using pictorial and physical mediators for linking the concrete and the abstract in mathematical understanding. This relates closely to the hierarchical modes or representation described by Bruner (1986), namely 'enactive', 'iconic' and 'symbolic'.

These modes refer to learning through a series of stages, firstly through physical action, then using pictorial images and finally through mental manipulation of symbols. Liebeck (1990) developed this idea further when she introduced ELPS, sequentially standing for Experience, Language, Pictures and Symbols. The close match with Bruner's earlier work is evident but adds the notion of the importance of language in learning.

Examples of the types of representations used in primary mathematics teaching include everyday objects such as boxes, coins, buttons and bottles, and then there are mathematical objects including linking cubes, plastic shapes and what is known as Base 10 apparatus. Finally there are pictures and diagrams including number lines and grids and images of everyday things such as purses. With regard to this study, trainees' choices of representations for their examples would be evidenced through their lesson plans and in how they describe in interview some of the examples in their mathematics lessons.

A second feature of the Transformation dimension is specifically the choice and use of examples for teaching and learning mathematics. It is recognised by Rowland *et al.* (2005) that trainee teachers in the United Kingdom will readily look to teaching manuals and the internet for lesson ideas and pupil exercises, leading to trainees' choice of examples as being a rich source for analysis. Examples themselves can be used for a range of purposes such as formation of concepts, procedural demonstration and as exercises for pupils. In this study, the examples chosen by trainees will be particularly examined in the light of the Transformation dimension.

In support of trainees' mathematical knowledge in the UK, the Primary National Strategy (PNS) (DfES, 2006) developed a pack of materials for use by trainees and their tutors which focused partly on the notion of choosing examples. The document is based on seminal research by Shulman (1986) who categorised knowledge for learning and teaching, and major contributors to the document included Thwaites, Rowland and Huckstep. Examples from their work are used in the PNS document for tutors and trainees to discuss, and these present a good scenario whereby the research outcomes based on types of knowledge are used to develop knowledge through the role of the tutor in higher education. It is still, however, up to the tutor to draw on their expertise to interpret the materials for their own trainees.

Thwaites, Rowland and Huckstep (2006 cited in DfES, 2006) describe three ways that examples fall short of being effectively selected for teaching, these being described as 'obscuring the variables', 'choosing a procedure that is not a sensible one' and 'random generation'. These types of examples are exemplified in the research as follows. When a teacher uses half-past six as an example of 'half-past' in teaching about time, the closeness of the hands on an analogue clock may cause some learners to be unclear about which hand is representing the 'half-past'. A non-sensible choice of procedure could occur, for example, when offering the calculation 203 – 199 and requiring children to calculate by a decomposition method, when counting on from 199 to 203 is more sensible.

The third type, the 'random choice' is often evident when teachers ask children to generate numbers for calculations or other tasks by rolling dice. This means that the numbers selected are not pedagogically chosen and could therefore either mislead learners or not enable them to develop concepts as well as they might if the examples were chosen more carefully. This work was developed by Rowland *et al.* (2009) who highlighted the issue of trainees' subject knowledge or lack of it in regards to their choice of examples.

Both types of examples described earlier, the inductive and those for illustration and practice, ought to be the outcome of a process of deliberate choice, according to Rowland *et al.* (2009). Inference of such a process was not possible from their evidence, but comment was made on the examples used and how they compare with available alternatives. Further cases of the types of examples to avoid are given by Rowland *et al.*, starting with obscuring the role of variables, where they offer the example of plotting (1,1) in the (x,y) plane, the obscurity being that since both the x-coordinate and y-coordinate are the same value, this is not helpful to learners who have yet to understand which coordinate is plotted in which direction and in which order.

Another such example is to add 9 to 9 on a 100 square, in which case the starting number 9 is the same as that which needs to be added, confusing a place on the 100 square with an action which is to move 9 places to get to 18. Similarly, adding 9 to 70 involves making a first move down a row and back to the start of the row before counting the remaining spaces to get to 79.

For an example of using one procedure when another would be more sensible, 49x4 by algorithm is offered, although this could more easily be calculated by using say, a doubling strategy twice, doubling being a key procedure in the National Numeracy Strategy. In summarising their work, Rowland *et al.* (2003a) report that novice teachers need guidance and help in appreciating the different roles of examples in mathematics teaching, and the existence of some common pitfalls in the selection of examples.

Thwaites *et al.* (2005) offer some insight into how the role of the tutor might be used to greatest effect for the benefit of the trainees by presenting a case study from their research on a group of PGCE trainees. After video recording a lesson taught by one of the trainees, she was interviewed about the lesson content, and in particular her choice of examples in working on complements of 100 and 1000. In starting with the example 82 + □ = 100, the trainee explained that the 82 was quite a random choice in terms of the digits except that she wanted to use a large number so the children only had a small complement to find. Her second choice of 35 + □ = 100 was deliberate in the sense that it focussed on a small number and therefore required a large complement. She might have considered that choosing 5 as the units digit may have made the calculation easier. What emerges from this episode is that the trainee began with arbitrary choices, but with reflection could articulate some thinking behind her choices and therefore demonstrates what Thwaites *et al.* (2005) describe as 'transformation' in that she moves from acquired knowledge to use of knowledge, an aspect of mathematics teaching that can be used as part of the training course, involving both tutors and trainees in reflecting on lesson episodes and identifying their uses of SMK.

In their conclusion, Thwaites *et al.* (2005) believe that their work will enhance tutors' understanding of trainees' subject knowledge as well as ways of engaging in reflective dialogue to support and build upon their existing subject knowledge.

Perhaps then 'learning by examples' is a property of the mind that has little in common with the pedagogical expectations expressed in the adage. An example is always embedded in a rich situation that contains more elements, data and information than just those directly related to the object exemplified. Sierpinska (1994) considers that the method of paradigmatic examples is not really a method of teaching, but rather a means of concept formation. The examples cannot be simply translated from the teacher's mind to that of the learner, and the learner must construct or re-construct examples that can be regarded as paradigmatic in a more objective sense. It might, therefore, be reasonable to suggest that to enable learners to access the qualities of any given examples in order to re-construct it for themselves, the teacher needs to ensure the example is one which will serve that purpose for the learner.

In summary, examples are an indispensable prop and a necessary obstacle to understanding abstract concepts. It is on the basis of examples that learners begin to make their first steps in concept formation, hence the crucial importance of providing appropriate examples as a teacher. If the intention is that the learner has a good range of examples to help them develop their concepts, teachers need to have a secure knowledge base of both subject and pedagogy in order to select appropriate examples.

### 2.4.3 The Connection Dimension

The third dimension in the Knowledge Quartet is that of Connection. In teaching mathematics, a number of elements need to be brought together for an effective lesson or sequence of lessons, such as planning, resources and examples. This dimension seeks to examine the coherence of those elements in terms of the sequencing of teaching 'based on knowledge of structural connections within mathematics itself, and an awareness of the relative cognitive demands of different topics and tasks.' (Rowland *et al.*, 2009, p.31) The connectedness of mathematics in a teacher's mind includes the sequencing of mathematical topics across a series of lessons as well as the order in which examples and tasks are used, and this aspect of Connection would be expected to emerge in this study as one of the ways in which trainee teachers demonstrate their choices for their lessons.

Pupil progress in primary mathematics, as measured by test score gains, was found to be associated with teacher beliefs and the nature of knowledge held. The Effective Teachers of Numeracy Project (Askew *et al.*, 1997) revealed that teachers who believed that all pupils could become numerate and who themselves had 'knowledge and awareness of conceptual connections between the areas which they taught' (p.3) produced the highest gains in numeracy tests for pupils. These so-called 'connectionist' teachers did not necessarily hold advanced qualifications in mathematics (A level or beyond), but were more likely to have attended extended CPD courses.

Initial teacher training was perceived to have little influence on effectiveness, raising questions about the amount of time devoted to mathematics on PGCE courses, and the relative contribution of ITT and CPD to deepen understanding.

In relation to the study by Askew *et al.*(1997), the team developing the Knowledge Quartet had at one stage decided to name one of their dimensions 'coherence' to distinguish it from the already popular phrase 'connection' in the Effective Teachers study. However, since this dimension related directly to codes from the research that related to how trainees sequenced their instruction, it seemed right that Rowland *et al.* should name the dimension 'connection' and whenever necessary provide an explanation of its derivation in their work.

### 2.4.4 The Contingency Dimension

The final dimension of the Knowledge Quartet is that of contingency. This dimension is one which covers a range of aspects of mathematics teaching that is either not planned or anticipated and how teachers respond to situations that arise unexpectedly in lessons. Whilst Rowland *et al.* (2009) argue that there might be fewer unexpected incidents in lessons when teachers have a better mathematical knowledge, it is what they describe as a teacher's ability to 'think on one's feet' (2009, p.32) that is categorised within contingency. Its two component aspects are teachers' readiness to respond to ideas that children put forward and in appropriate cases, their readiness and ability to deviate from their set plans in order to accommodate the opportunities put forward by children.

An example of this type of teacher behaviour is documented by Bishop (2001, p.244) where he recalls an occasion when a class of Year 5 children were learning about fractions and were asked to give an example of a fraction between ½ and ¾. One girl responded by suggesting 2/3, since (comparing numerators) '2 is between 1 and 3, and on the bottom [comparing denominators] the 3 lies between the 2 and the 4.' The solution given by the child is correct, but the teacher, Cathy, could not be sure about the reasoning. She then faced a dilemma in the middle of her lesson: should she accept the answer without challenging its validity, should she question the child to verify whether such a process always works, or should she deviate from the next planned part of her lesson to spend time with the class exploring properties of fractions which are formed in the manner described by the girl? Her choice of strategy at this point will depend upon a number of factors, among these being, as Rowland *et al.* (2009, p.131) describe, 'the mathematical specialised content knowledge that Cathy possesses, both substantive and syntactic.'

In the episode with Cathy and the fractions, the child may have had misconceptions about how fractions can be manipulated, and in other topics within mathematics, children often give incorrect responses to questions because of misconceptions. Skilful teachers will use exposed misconceptions to improve children's learning and Koshy (2000) describes ways of using errors and misconceptions to improve learning.

The importance of the Contingency dimension is the link between the teacher and the child, since it is the informed and knowledgeable intervention on the part of the teacher that will contribute to the child being able to extend its knowledge and make appropriate connections between prior and new learning. In this study, it is unlikely that any evidence of contingency will emerge from the analysis of lesson plans, however, interviews with trainees about their teaching may reveal that their knowledge and choices of example leave them vulnerable to incidents where a readiness to respond and deviate are necessary.

## 2.5 Conclusion

This chapter has set out to examine current and recent literature which supports and gives a context to the study of primary trainee teachers' choice of mathematical examples. By considering categorisations of examples and studies which have brought to light a number of aspects of trainees' choices, the basis has been laid against which the research data could be analysed. To provide a background to the trainees' context, aspects of the statutory and non-statutory curriculum guidance has been considered and together with research findings on examples and their use, the review of literature provides a conceptual framework for this study. The key literature for the research will be the work of Rowland *et al.* (2009) on the Knowledge Quartet, Watson and Mason (2005) on examples, with Bills and Bills (2005) on trainees' use of examples and Shulman (1986) on types of knowledge for teaching also contributing to the research design and subsequent data collection and analysis.

From the literature, the formulation of three research questions came about:

1. What pedagogic considerations are used by a cohort of trainee primary teachers when choosing mathematical examples in the classroom?
2. How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?
3. Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?

The key ideas which emerged from the literature as being significant in relation to this study are therefore:

- Planning procedures for teaching mathematics (DfEE, 1999)
- Resources that were used in planning and teaching (Haylock, 2006)
- Relationship between choice of examples and children's learning (Watson and Mason, 1998)
- Consideration of examples in published materials (Bills *et al.*, 2006)
- Factors involved in the choice of examples (Watson and Mason, 2005)
- Relationship between subject knowledge and choice of examples (Rowland *et al.*, 2009)

These questions and key ideas are based on the literature around knowledge of mathematics SMK or mathematics pedagogy, since this may inform choice of examples. Examining these for the sample group in this research will help answer the first two research questions, relating directly to the strategies for choosing examples, whilst the third research question focuses on the effect of mathematical subject knowledge on trainees' choice of examples.

# Chapter 3 Methodology

## 3.1 Introduction

This chapter commences with a consideration of the methodological design for the research, and outlines why a particular approach to methodology and methods has been used. It will then explore in more detail the specific methods associated with the selected methodology and describe how these methods were adopted in this study, along with any difficulties that arose and how they were overcome.

The use of case study as a research design appropriate for examining different primary teacher trainees' approaches to choosing mathematical examples will be outlined, including a consideration of the positive and negative aspects of case study as a choice of research design. Methods of data collection are then explored in detail, with reflection on the researcher's role as an instrument of data collection and the relevance of selection procedures are discussed in accordance with the study's aims. Finally, a general analytical procedure is set out that will facilitate the examination of teaching and learning in mathematics, with specific focus on the choice of examples.

### 3.2 Choice of Methodology

Central to the research design is the focus upon 'what information most appropriately will answer specific research questions, and which strategies are most effective for obtaining it' (Le Compte and Preissle, 1993, p.30). From interviewing a sample of primary trainees, it is now understood that:

> There is no clear window into the inner life of the individual. Any gaze is always filtered through the lenses of language, gender, social class, race and ethnicity. There are no objective observations, only observations socially situated in the worlds of the observer and the observed...No single method can grasp the subtle variations in ongoing human experience.
> (Denzin and Lincoln, 1998, p.24)

While recognizing, as Denzin and Lincoln (1998) do, that researching other people's lives is a complex task, a case study design has been adopted, which acknowledges this complexity by deploying 'a range of interconnected interpretive methods', to illuminate and 'to make more understandable the worlds of experience that have been studied' (Denzin and Lincoln, 1998, p.24). The everyday interactions between people in different settings is most effectively researched using ethnomethodology, an approach which can be traced back to the work of Garfinkel (1967), for example. It is a method which studies the methods that individuals use to make sense of their social world and accomplish their daily actions. In this way, ethnomethodology is the study of the systematic processes used by individuals to negotiate their way through everyday actions. It focuses on uncovering the rules that direct ordinary life, could be applied to teachers or students and the choices they make in teaching mathematics and is concerned with how things happen, rather than what happens and why it happens.

Due to this focus, ethnomethodology could be considered an appropriate methodology for this research, although it was not the dominant approach.
The exploration of cultural groups is designed to acquire a better understanding of populations or individuals. Social scientists often rely on survey or interview processes to build understandings of such groups, and may inquire of a number of group members in a bid to understand their attitudes, beliefs, opinions and behaviours. However, probing deeper into cultures may lead beyond an exploration of simply what is, and may begin to explain why it is. The dominant research methodology for exploring cultural groups is ethnography, which involves interpreting the way of life of the participants within the group.

Such groups are usually linked by more than genetic or biological factors and are more often bound together by social traditions and common patterns of beliefs and behaviours, such as those exhibited in teachers and those training to become teachers. In the context of this research, it is not the group practices that are significant, but those of individuals within a defined population and so the ethnographic approach is dismissed as being inappropriate.

The subjects for this research are linked by virtue of the fact they are all training to be primary school teachers at one higher education institution, and all are required to follow a programme which provides them with knowledge, skills and understanding associated with teaching mathematics. They form a bounded system, that is, a number of particular instances or entities that can be defined by the identifiable boundaries of institution, course and subject.

The exploration of such a bounded system can be described as a case study, and where a number of the particular instances are explored, a multiple case study is generated. In the current research, a sample group was used consisting of trainees from two cohorts whose individual understanding and application of mathematical examples for teaching were explored, with the trainees themselves forming the multiple cases from which observations and descriptions were made.

### 3.2.1 The Case Study

This section considers in more depth the case study as being a research design that is most appropriate in examining individual practice in teaching mathematics as a primary trainee teacher. The positive and negative aspects of such a choice are considered and explanations are given as to how significant criticisms have been addressed. Case studies can take on various forms and be used in different contexts in fields such as medical, social work and psychology, but in relation to this study it is defined as qualitative educational research (Yin, 1994). The case study is an attempt to gather in-depth data, so that a specific instance or phenomenon may be understood (Robson, 1993; Yin, 1994; Stake, 1995) through the deployment of a variety of research methods. Though these methods are often considered to be common to the case study, criticisms have been made because of the way in which the case study has been defined by the methodology rather than the focus being put upon the object that is being studied (Stake, 1994).
Other criticisms suggest that it provides little basis for generalization beyond the case itself, and that the data lacks credibility because of the lack of rigour applied in collation, resulting in doubtful evidence and researcher bias (Yin, 1994).

The case study is initially defined as the object or instance that has been chosen to be studied (Stake, 1994), which, in relation to this study, is the examination of planning documentation for primary mathematics from trainee teachers on their final school placement, and their consideration of the examples they use in their teaching. It is based on a naturalistic paradigm with a phenomenological orientation: the research examines the many different influences that contribute to the selection of mathematical examples in a real teaching context, with particular focus on the way trainee teachers use theory and teaching resources to identify and select appropriate examples for their teaching. If the 'dimensions of research provide a "road map" through the terrain that is social research' (Neuman, 2000, p.39), then it could be concluded from this initial definition that the case study marks out a road in the research process by including elements that are generic to research design as a whole through design, data collection and analysis, recognition of problems and writing (Yin, 1994).

### 3.2.2 Types of Case Studies

In using the term case study, Stake (1994, p.236) argues that some may use it because it draws attention to the question 'What can be learned from the single case?' This is intrinsically linked to the research questions which underpin the study as a whole and which ultimately define the type of case study chosen.
It is important for the purposes of this study to define clearly the type of case study used, so avoiding ambiguity, and to justify reasons for methodological choices. Stake (1995) defines three types of case study in order to show how researchers study cases for different purposes.

They are the intrinsic case study, the instrumental case study and the collective case study. Stake (1994) warns that these are not prescriptive labels that case studies neatly fit into but are 'heuristic' in nature, so allowing discovery rather than dictating a function.

The intrinsic case study is undertaken solely with the interest of the case in and of itself, rather than having external aims or objectives as the motivating factor of the study. The instrumental case study is used to highlight a particular issue or theory so the case becomes the facilitator by which the external aims and objectives of the study are achieved. Thirdly, the collective case study is the study of multiple cases, which enable knowledge to be gained about a certain phenomenon or group population to enable wider generalizations to be made from representative cases.

The seven cases chosen from the sample group of primary trainee teachers demonstrate the instrumental aspect of case study research in their focus on mathematical subject knowledge and its relation to trainees' choices of mathematical examples. The collective aspect of case studies is shown through the commonality of the cases in terms of their location and training.

### 3.2.3 Case Study Methods

Careful thought was needed to determine which methods are most appropriate in capturing the complexity of planning to teach mathematics and the choice of examples this entails. Stake (1994) describes differing methods for instrumental and collective case studies.

The former focuses on issues raised by other researchers, whilst the latter seeks to draw out new theories. Stake (1994, p.237) summarises the characteristics of the instrumental case study as:

> A particular case is examined to provide insight into an issue or refinement of theory. The case is of secondary interest; it plays a supportive role, facilitating our understanding of something else...The choice of case is made because it is expected to advance our understanding of that other interest.

As the focus of this study is the choice of mathematical examples for teaching by trainee primary teachers, it is the instrumental nature of the cases that are reflected in the methodologies chosen. The problem with focusing upon the uniqueness of a single case is the reduction of opportunity to generalize beyond the setting of that case. This is a criticism of the case study design (Denzin and Lincoln, 1998; Punch, 1998; Bryman, 2001), but Stake (1998) warns that details of case studies can be overlooked because of the desire to make generalizations.

While it seems clear there are dangers in generalising from distinct case studies, Stake (2000, p.22) suggests that knowledge gained from an understanding of the individual case can be open to naturalistic generalisation: 'arrived at by recognizing the similarities of objects and issues in and out of context and by sensing the natural covariations of happenings.' Indeed, the application of Stake's (1994) instrumental and collective case study designs encourage generalisability through reference to relevant theoretical constructs and comparative analysis.

Lincoln and Guba (2000, p.38) argue that:

> While the idea of naturalistic generalization has for us a great deal of appeal,... we do not believe that it is an adequate substitute or replacement for the formulistic or logical generalizations that people usually have in mind when they use the term 'generalisation'.

Further, they use a term developed by Cronbach (1975), the working hypothesis. Cronbach states (1975, p.125): 'When we give proper weight to local condition, any generalization is a working hypothesis, not a conclusion.' Lincoln and Guba (2000) suggest that generalizations should come later, if at all, and that if one situation is to be compared with another then both contexts need to be understood before an appropriate judgement can be made. Stake (1994) also acknowledges the need to understand the complexities of the individual case before it can be placed in a wider context.

This study is not large enough to enable reliable generalizations about the wider population of United Kingdom initial teaching trainees, but encourages, as Stake (2000) suggests, naturalistic generalization to take place as a result of readers constructing their own interpretations and recognizing commonalities in the case studies. Through the collection and analysis of data, this research attempts to capture the uniqueness and commonalities of each particular case and present them as 'working hypotheses', proposed points of generality which may be transferable when making cross-case conclusions (Yin, 1994). In developing the cases in this study, themes from lesson plan analysis and transcribed interviews were used to characterise trainee teachers' ideas about planning mathematics and choosing examples. By engaging in cross-case analysis, it was possible to examine the cases for similarities and differences between the trainees.

### 3.3 Methods of Data Collection

To gain access and insight into the choice of mathematical examples by trainee primary teachers and the resources and planning methods they adopt in making such choices, access was needed to relevant data which were recorded using the most appropriate methodological tool. The methodological processes involved are now discussed to demonstrate how this was achieved in a case study design. The methodology is recorded by introducing each tool discretely. After consideration, it was felt that a chronological approach would be less appropriate and realistic, as this did not reflect the way methodological tools had been used according to the context in which they were needed throughout the data collection period.

This demonstrates, as Denzin and Lincoln (2000, p.22) suggest, that a research design 'describes a flexible set of guidelines that connect theoretical paradigms first to strategies of inquiry and second to methods for collecting empirical material'. This flexibility allows the portrayal of the methodology process in a way that is most realistic and helpful while keeping the flow of narrative, so that, as Stake suggests (1995, p. 134) 'The report may read something like a story', still within discrete categories of data collection. The role of researcher played a central part in the process, particularly in the way personal traits were employed to obtain data. Janesick (2000, p.389) notes:

> Because qualitative work recognizes early on the perspective of the researcher as it evolves through the study, the description of the role of the researcher is a critical component of the written report of the study.

From the outset of the study, the researcher's background influenced the research process. As a mathematics educator from primary teaching, there was no difficulty in being accepted as part of the professional culture by the participants. Further study by the researcher at Masters Level and ongoing involvement in the field of mathematics education affected the study from a positive personal perspective, initially by helping determine the topic to study. The data collection methods used with the case studies were:

- Collection and analysis of mathematics lesson plans
- Interview of a sample of trainees
- Audio recording, transcribing and analysis of interviews
- Collection and analysis of classroom resource materials

### 3.3.1 Methods for each cohort

The following methods were chosen as the most appropriate in collecting rich material that would illuminate the complexity of mathematics planning, and are discussed in more detail in the remaining sections of this chapter.

Phase I – 2005-08 cohort

- Interviews with a sample of 10 trainee primary teachers;
- Initial analysis of over 400 mathematics lesson plans from 22 trainees;
- Detailed analysis of the lesson plans and interview transcripts from four trainees.

Phase II - 2006-09 cohort

- Interviews with a sample of 15 trainee primary teachers;
- Initial analysis of over 300 mathematics lesson plans from 18 trainees;
- Detailed analysis of the lesson plans and interview transcripts from three trainees.

### 3.3.2 Choice of Instruments for Data Collection - Interviews

The interview was chosen as a tool for data collection because, as Robson (1993, p.229) suggests, it 'has the potential of providing rich and highly illuminating material', accessing the beliefs, values, meanings and perceptions of research participants that underpin their actions. Whilst there is a risk of bias and concerns over reliability, these can be overcome by employing reflexivity and data triangulation, so that interpretations are made from a firmer evidential base. Interviews demand skilful preparation and practice, but it was felt that the time and effort was worth investing because of the power of discovering participants' understanding of themselves in the mathematical teaching environment.

Interviews played a vital role in the study. They were designed to encourage discussion about reflection and issues related to the choice of mathematical examples. An important aspect of the interview is its adaptability in the generation of data. The choice of interviewing as a research method supports an ontological position which holds that people's knowledge, views, understandings, interpretations, experiences and interactions are all meaningful properties of the social reality which the research questions are designed to explore (Robson, 1993).

### 3.3.3 Semi-structured Interviews

Interview types can range from formal through semi-structured to unstructured. A formal "structured" interview involves the interviewer being as consistent as possible. In this type of interview one must try to create a copy of the situation and questions used in previous interviews or interviews to come. A structured interview is often characterised by the use of a questionnaire or a checklist. This is not appropriate for the case studies as the research needed to explore issues that arose through discussion. The unstructured interview generally tends to centre on a topic. A semi-structured approach was chosen because the interview needed a focus and the interview questions would create data that were consistent and could be linked to the literature.

Semi-structured or informal interviews as defined by Robson (1993) were conducted with the student cohorts to provide rich, descriptive data on how they select examples for teaching within the context of planning and how they perceive the importance of their choice of examples. The purpose of the semi-structured interview is that though it still has a specific research role in gaining access to situations (Burgess, 1984), the nature of that role is more implicit with a structure open to modification depending on the 'conversation'. With a conversational style the semi-structured interview appears to be, and may feel, more natural and easier to conduct.

Some researchers polarise structured and unstructured interviews, suggesting that the unstructured interview is conversational while the structured is not (Burgess, 1984), Holstein and Gubrium (1997, p.113) argue that all forms of interviews are conversational: 'Interviews are special forms of conversation. While these conversations may vary,... all interviews are interactional'. Whilst the interview is a form of conversation, it should be recognised that it is different from social interactions that take place everyday between individuals (Robson, 1993). The interviews took the form of: 'a conversation with a purpose' (Robson, 1993, p.228) and the intention was that all the interviews should be conducted in an informal and open way to engage with participants in obtaining the required data.

Crucial to these conversations were the nature of the relationships between the participant and the interviewer, for the quality of data can be affected by the way in which the interviewer and interviewee relate. Listening with interest to what the respondent has to say is one of the most important skills employed in the interviewing technique (Punch, 1998), thereby showing interest and genuine curiosity about what they are saying (Burgess, 1984; Mayhut and Morehouse, 1994). It was relatively easy to identify with trainee primary teachers through having had similar experiences as a primary teacher and knowledge of the training as a former tutor on the course, thereby gaining insight into the context of their situations. Listening is an essential part of a dynamic and active dialogue between interviewer and interviewee. However, it is important that the researcher shows expression when the interviewee is talking, to aid openness and to remain alert to what is being said in relation to key phrases and terminology that might need further probing.

In preparing for an interview the topics were selected, questions were written, the method of analysis considered and a schedule prepared and used. After the interviews, evidence from lesson plans and from transcripts of the interviews was examined in order to make judgments about the reflection that the subjects engaged in. To strengthen the validity of any conclusions made, a process of respondent validation was used where subjects were asked to comment on the conclusions that were made. These comments were taken into account when redrafting conclusions made as a result of the interviews.

### 3.3.4 Choice of documents

Specific documentation was collected to ascertain its purpose in the context of supporting trainees in their choice of mathematics examples: the Mathematics National Curriculum, the Primary National Strategy Framework for Mathematics, lesson plans and resource materials, both published and home-made. In choosing to use documents as data sources, methodological issues needed to be considered. One of the attractions of using documents in research is that they may have been written 'live', catching the dynamic situation for the context in which they are set. However, as Cohen *et al.* point out, 'they may be highly biased and selective, as they were not intended to be regarded as research data but were written for a different purpose, audience and context' (2007, p.201). Documents are also what Robson describes as an 'unobtrusive measure', since the documents themselves are neither altered or affected in any way as a result of being used and analysed (2002, p.349).

In this study, the first issue is one of access to lesson plans for mathematics lessons, and these are generated as documents as part of the requirements of trainees for their course. In gathering information about the lesson intentions, copies of lesson plans were available directly from the trainees who formed part of the research sample and so access was on a voluntary basis by each trainee. The documents contained a range of information, much of which was not relevant to this study, but in most cases, the detail for each lesson contained a range of typical examples that the trainee would use in their direct teaching.

A second issue with the use of lesson plans is that whilst they are designed to be used as a set of intentions for teaching and learning, between writing the plan and teaching the lesson, the trainee may make planned or unplanned alterations, either by annotating the document shortly before using it, or by altering the teaching during the lesson which will not necessarily be reflected on the plan, being more likely to be recorded as part of the lesson reflection in a different document.

### 3.3.5 Triangulation

In the collection of data one of the major strengths of the case study design is perceived to be the multi-method or triangulation approach (Denzin, 1978; Williams, Karp, Dalphin and Gray, 1982). This allows the cross-checking of data to guard against researcher bias and to present a fuller and more comprehensive picture of the phenomenon being studied. By using multiple triangulation in the methodology the reliability and validity of the study can be strengthened (Hakim, 1987; Robson, 1993; Silverman, 2001).

One of the difficulties of this approach is the integration of the multiples of data collected (Burgess, 1982a), as different accounts of the same situation result in a complex description of subjects and events (Burgess, 1984; Neuman, 2000). Sayer (1992, p.223) suggests that this can be resolved if: 'The meaning of each part is continually re-examined in relation to the meaning of the whole and vice versa', achieving greater credibility to the evidence of the individual accounts and the narrative as a whole. While methodological triangulation can still lead to data being misinterpreted by the researcher, Shipman (1997, p.106) suggests that: 'Triangulation is an acknowledgement that social research is rarely decisive and that confidence is often best established by collecting and presenting a number of viewpoints.'

### 3.4 Selection of Cohorts

The choice of students from two separate cohorts was necessary from two key perspectives. Firstly, the use of an initial cohort followed by a second distinct cohort allowed the development of research skills in refining the selection process and the interview schedule for the data collection as a part of gaining valuable research experience. Secondly, since the students were invited to volunteer to be interviewed and to provide lesson plans for analysis, there was no guarantee that a sufficient number of students would come forward from which to draw a suitably representative sample. This being the case after the first year of data collection, it was decided to expand the sample group of students by inviting those in the following cohort to also take part in the research.

This second cohort then provided sufficient students from which to obtain a representative set across the B.Ed programme and to then select case study students from the combined sample group over both cohorts.

In order to research trainees with little classroom experience but close to completing their training, the cohort of Year 3 trainees from the Bachelor of Education programme over two consecutive academic years were chosen. These trainees were admitted to the course having attained a minimum grade 'C' in mathematics in the General Certificate of Secondary Education (GCSE) and achieved at least 40% in a 10-item test of mathematical attainment taken as part of the interview procedure for securing a place on the course. The cohort from each year who had reached Year 3 of the course consisted of approximately 120 trainees, most of whom had completed the first two years of the course without need to repeat a year due to, for example, failing modules or personal issues. A small proportion of each cohort had either transferred into the course after Year 1 from another university, or was in the final year but started more than two years previously due to deferment or failed modules. After consideration of various measures of mathematical attainment, to be explained in the following section, the intention in the research design is to divide the sample group into three categories of attainment.

### 3.4.1 Pre-Course Data

Pre-course data collected on the trainees in the two cohorts represents their mathematical attainment measured in a number of ways. This is a relevant parameter for studying trainee performance since the link between mathematical subject knowledge and choice of mathematical examples in teaching is being researched.

The first set of data is their exam grade in mathematics at GCSE (General Certificate of Secondary Education), which is usually taken at the end of Key Stage 4, when pupils are in Year 11 and aged 15 or 16 years (Appendix 11a and 11b). The grades collected give an indication of the attainment range of the trainees at that level, with the top grade being A*, then A, B and C. Lower grades from the GCSE exam are not considered, since a minimum of grade C is required by trainees to obtain a place on the Bachelor of Education programme. A small proportion of the trainees will have studied either AS or A-level mathematics after completing the GCSE course, and the grades for those trainees are included as an indication of their further attainment in the subject (Appendix 12a and 12b).

After a trainee has applied for a place on the course, they attend for interview. This process includes tests in mathematics and English, as well as a presentation task and both group and individual interviews. The mathematics test is a timed test covering number, algebra, measures and problem solving and the scores for this test are shown in Appendix 13a and 14a.

A pass mark of 40% is required, and for those trainees who are offered a place but did not achieve the pass mark, they are required to retake the test. The marks from the retaken tests are also shown in Appendix 13b and 14b. The tables show that two students in the cohort achieved 100% on the interview test, and that both these students are part of the sample group for Phase I of the research. These students are clearly interested and able in mathematics, and so are perhaps more likely to volunteer to be part of the research sample. The sample group therefore could be biased towards the more able students in the spread of attainments.

Once admitted onto the course, all primary teacher trainees undergo a diagnostic numeracy test within their first two weeks, which covers seven areas of mathematics:

- Basic arithmetic
- Percentage and ratio
- Basic algebra
- Problem solving
- Significant figures and indices
- Charts, tables and graphs
- Fractions and decimals

The test is given in a booklet of 77 questions, with topics mixed throughout the test. The test is in multiple choice format with four possible answers, only one of which is correct, and one response which is 'other'. For each area, there are questions offering a maximum of 11 marks, one mark per question, and any trainee who scored poorly (less than 7 marks) on four or more areas was offered the opportunity of taking an additional module in Year 1, 'Confidence Counts', which is designed to improve their mathematical knowledge and understanding.

The diagnostic test scores that are available are given in Appendix 15a and 15b, although these are not the full set of cohort results as it was not possible to obtain permission to access every student's result of the diagnostic test.

### 3.4.2 Module Assignments

During the course, the trainees take three mathematics modules which cover aspects of teaching primary mathematics such as planning, assessing and subject knowledge. In Year 1, the module aims to develop each student's personal mathematical subject knowledge and begins to develop their knowledge about pedagogical issues relating to the teaching of mathematics in school. The taught module and school placements enable the students to develop insight into the way mathematics is learned.

The delivery of the course is differentiated according whether trainees are following the programme for teaching Foundation Stage and Key Stage 1 (age 3 – 7 years) or the Key Stage 1 and 2 (age 5 – 11years) and this is reflected in each of the modules from Year 1 onwards. The course consists of lectures and workshops as well as group work, presentations and work on mathematics problems. The Year 1 module focuses upon the teaching and learning of number including counting and the number system, algebra, functions and graphs. This is addressed through problem solving and investigative work. To be assessed on the module, students submitted a journal, in two parts, showing their work on a number of mathematical problems during the module.

This work demonstrates the development of their understanding of investigating and problem solving in mathematics, and subject knowledge at a level of mathematics that challenges the students' personal levels of understanding.

Students write a summary for the assessment, reflecting on the development of their mathematical understanding in terms of:

- The mathematical topics covered by the problems
- The development of mathematical subject knowledge
- The key mathematical results of the work
- What they have learnt about problem solving
- Some analysis of the different stages and processes of mathematical investigation in their problem solving
- Identification of whether they were able to prove their conjectures
- How their thinking about mathematics as a subject has developed.

In Year 2, the emphasis of the course moves from developing subject knowledge *per se* through problem solving and investigational techniques to learning more about teaching mathematics in school. The module develops students' ability to teach number, algebra, data handling, measures, 'shape and space' and 'using and applying mathematics'.

The students develop their knowledge of the National Curriculum (DfEE, 1999) and Primary National Strategy (DfES, 2006) and use their knowledge to organise and assess the mathematical learning of children. The module also covers the use of information communication technology (ICT) in preparing the students to plan, teach and assess mathematics in school. The purpose of the assignment in the Year 2 module is to practise teaching number in a realistic context and then to demonstrate understanding of the mathematics involved and its teaching and learning through reflection and evaluation.

In Year 3, the mathematics module develops students' ability to critically analyse and reflect upon broader aspects of mathematics education. This involves analysis of research, Ofsted and Primary National Strategy reports and the link between mathematics and other curriculum subjects. The assessment of the Year 3 module involves planning a teaching episode using an 'I can...' statement from the Primary National Strategy. The purpose of this is to demonstrate understanding of teaching and learning mathematics for either 'shape and space' or 'measures'. It also helps students become more familiar with the Primary National Strategy. Students identify the mathematical subject knowledge (facts, skills and concepts) they would be teaching and the prior skills, knowledge and understanding they assume the children already know that would be a relevant pre-requisite for this objective. They are also asked to discuss critically the examples they chose to model the concepts. The module results for the cohorts, where available, are shown in Appendix 16a and 16b.

### 3.4.3 Invitation to cohorts to take part

All students in their final year at the time of Phase I of the data collection were contacted during spring 2008 inviting them to take part in the study. The context of the research and the data collection process were explained in a letter sent by email, along with a sample consent form to be used if trainees were willing participants. The information was sent shortly after the trainees had completed their final school placement and so there was no possibility that trainees taking part had prepared materials specifically for the research.

This may have affected data in Phase II when trainees were informed about the research ahead of starting their final placement in spring 2009. All students in their final year at the time of Phase II of the data collection were contacted to invite them to take part in the study.

The context of the research and the data collection process were explained in a letter sent by email, along with a sample consent form to be used if trainees were willing participants. The information was sent before the trainees started their final school placement and so although it was possible that participant trainees had prepared materials specifically for the research, in practice they were focused on writing lesson plans to teach whilst on placement and on being assessed by the school and their placement tutors.

The information given to the trainees explained that mathematics lesson plans were to be collected from as many trainees as possible, and following collection problems the previous year, it was decided that trainees could bring or send their lesson plans as soon as was convenient after the end of their final placement.

### 3.5 Collecting the Data

In this section, the practical details on how the data were collected are described, along with the difficulties that arose and how these were overcome. This will include the logistics of gathering lesson plans from the two cohorts of trainees and the subsequent arrangements for interviewing those who had volunteered.

### 3.5.1 Lesson Plans

For Phase I of the data collection process, information given to the trainees explained that mathematics lesson plans were to be collected from any willing trainees, and due to the logistics of the course programme during the final term, it was arranged to collect the plans when the cohort were attending their final lecture. The location of the main lecture theatre provided sufficient room in the adjoining lobby for boxes of plans to be collected as the trainees arrived. The options offered to the trainees were that they could provide copies of any plans they did not need returned, or they could provide originals which could be copied and returned.

On the collection date, many trainees seemed more focused on submission of their dissertations after the lecture than on remembering to bring examples of mathematics lesson plans for this study. Some students apologised for not remembering, while most walked by without comment. Some said they would bring plans at a later date. Over the following month, 22 trainees brought a collection of plans to be used as data. This was around 400 separate lessons, covering a range of pupil year groups and mathematics topics. In Phase II of the research, 18 trainees brought around 300 separate lessons to be used as research data, covering a range of pupil year groups and mathematics topics.

### 3.5.2 First Analysis of Lesson Plans

After collecting lesson plans in Phase I, a table was drawn up (Table 1) to sort the plans by school age group and by mathematics topic to ascertain which year group had most representation amongst the plans, and which mathematics topics were the most common. By separating the year groups and topics, it was possible to focus the analysis of examples to those year groups and topics which offered most data, by selecting as cases those trainees who had taught those topics and in those year groups. The sorting demonstrated that Year 3 and Year 4 lessons were most commonly represented in the primary range, followed by those from the Reception year. Trainees tend not to have placements in Year 6 classes so that school preparation for Standard Assessment Tasks (SATs) at age 11 is not interrupted. The most common topics were addition and subtraction, which accounted for 66 plans, followed by 52 on multiplication and 42 on 2D and 3D shape (Appendices 17a and 17b).

**Table 1: Numbers of lesson plans provided by trainees from 2005-08 cohort, sorted by topic and year group**

| Year<br>Topic | R | R1 | 12 | 2 | 23 | 3 | 34 | 4 | 45 | 5 | 6 | Totals |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2D & 3D shape | 10 | 6 | | 8 | | 2 | 5 | 6 | | | 5 | 42 |
| Symmetry | | | | 4 | | 4 | | 3 | | | | 11 |
| Time | 3 | | | 1 | | 2 | 2 | 5 | 2 | 1 | | 16 |
| Angles | | | 2 | | 3 | 2 | | 8 | | | 2 | 17 |
| Compass directions | | | | | 1 | 1 | | 7 | | | | 9 |
| Number bonds | | 3 | 3 | 4 | | 3 | | | | | | 13 |
| Multiplication | | | 2 | 10 | 3 | 5 | 2 | 7 | 9 | 10 | 4 | 52 |
| Division | | | | 1 | 5 | 3 | 1 | 4 | 3 | 7 | | 24 |
| Word problems | 1 | 1 | 3 | 9 | 9 | 5 | 4 | | | 1 | 1 | 34 |
| Addition/Subtraction | 13 | 5 | | 4 | 8 | 6 | 8 | 21 | | 1 | | 66 |
| One more/less | 4 | 4 | | | | 1 | | | | | | 9 |
| Counting/Rounding | 13 | 7 | | 4 | | 2 | | | | | | 26 |
| Measures | 2 | 4 | 2 | | 8 | 10 | 6 | 1 | | | 1 | 34 |
| Area/perimeter | | | | | | | 1 | 5 | | 8 | 1 | 15 |
| Sequences | 3 | 4 | 2 | 8 | | 3 | | | 5 | | 1 | 26 |
| FDPR | | | 5 | | 5 | 5 | 1 | 11 | 9 | 2 | 2 | 40 |
| Calculations | | | | | 1 | | 1 | | | | 2 | 4 |
| Algebra | | | | | | | | | | | 1 | 1 |
| Data Handling | | | | 4 | | 3 | 10 | 1 | | | 1 | 19 |
| Probability | | | | | | | | | | | 2 | 2 |
| Coordinates | | | | | | | | | | | 1 | 1 |
| **Totals** | 49 | 34 | 19 | 57 | 43 | 57 | 41 | 79 | 28 | 30 | 24 | **461** |

From the collection of lesson plans in Phase II of the data collection, a table was drawn up (Table 2) to sort the plans by school age group and by mathematics topic in order to ascertain which year group had most representation amongst the plans, and which mathematics topics were the most common amongst the set of plans. By separating the year groups and topics, it was possible to focus the analysis of examples to those year groups and topics which offered most data. This sorting demonstrated that Year 3 and Year 3/4 lessons were most commonly represented in the primary range, followed by those from Year 5.

**Table 2: Numbers of lesson plans provided by trainees from 2006-09 cohort, sorted by topic and year group**

| **Year** | **1** | **12** | **3** | **34** | **4** | **5** | **6** | |
|---|---|---|---|---|---|---|---|---|
| **Topic** | | | | | | | | **Totals** |
| 2D & 3D shape | | 3 | 17 | 8 | | 2 | | 30 |
| Symmetry | | | 5 | 3 | | | 1 | 9 |
| Negatives | | | | | 1 | | | 1 |
| Reading scales | | | 2 | | | | | 2 |
| Angles | | | | 2 | | | | 2 |
| Directions | | 3 | 2 | | | | | 5 |
| Coordinates | | | | | | 3 | | 3 |
| Multiplication | | | 9 | 6 | 7 | 3 | | 25 |
| Division | | | 3 | 3 | 2 | | | 8 |
| Word problems | 1 | | 2 | 5 | | 3 | 1 | 12 |
| Addition/Subtraction | 4 | | 30 | 9 | 9 | 6 | | 58 |
| Inverse Operations | | | | 1 | | | | 1 |
| Measures | | | | 1 | | | | 1 |
| Area | | | | | | 3 | | 3 |
| Mass | | 4 | | 2 | 2 | 1 | 1 | 10 |
| Length | | | | 5 | | | | 5 |
| Capacity | | | 7 | 4 | 1 | | | 12 |
| Fractions | 3 | 2 | 10 | 24 | | 1 | | 40 |
| Decimals | | | | | | 2 | | 2 |
| Time | 3 | | 11 | 1 | | 5 | | 20 |
| Problem solving | | | 1 | 3 | 2 | | | 6 |
| Data Handling | | 5 | 10 | 9 | 2 | 2 | | 28 |
| Place Value | | | | 3 | | | | 3 |
| Money | | 3 | 4 | 1 | | 2 | | 10 |
| Sequences | 1 | | | 5 | - | | | 6 |
| **Totals** | 12 | 20 | 113 | 95 | 26 | 33 | 3 | **302** |

In terms of topics represented, this cohort provided 58 lesson plans on addition and subtraction, 40 plans on fractions and 33 on multiplication and division. (Appendices 18a and 18b)

The final composition of students for the sample group and the subsequent case studies became determined to some extent by the availability of lesson material. The research therefore focused on those school year groups and mathematical topics which were most prevalent amongst all those collected. This will inevitably introduce a degree of bias into the work, acting as a constraint to possible interpretations across the whole mathematics curriculum, but conversely this approach provides a focused approach on a significantly important area, namely that of number.

As well as lesson plans, resource materials such as worksheets were collected, both published and 'home-made' by either the trainees or their placement class teachers. The published resources tended to come from a limited range of materials, with only a small number of educational publishing companies being used. Amongst the material from publishers are mostly pupil textbooks, with pages allocated to different topics, demonstrating particular concepts by way of a range of examples. These examples form part of the analysis. Self-produced worksheets are generally variations of those found in published materials, and modified to suit the particular needs of a teacher and their pupils. The modifications are most often based on the notion of differentiation of tasks for the attainment range of the group or class for which the worksheet is designed.

### 3.5.3 The interview process with the trainees

Having identified the trainee teachers from the respective cohorts, interview dates were arranged with each participant so that data could be collected about how they approach mathematics teaching and planning, how they choose examples for teaching and draw on their experiences from their completed final school placement. Interviews were held with the trainees during the summer term of their cohort's final year. The dates for interviews were agreed to enable the trainees to be available and took place, in some cases, in the university refectory, whilst most were held in the researcher's office which offered a quieter and undisturbed location.

Before each interview, the participants were given an explanation regarding the purpose of the interview (repeating information previously received in printed form), their right to withdraw at any point, their right to a printed copy of the interview transcript to check for accuracy. Their permission was sought for the interview to be recorded using a digital voice recorder and interviews lasted between 20 and 50 minutes.

Careful consideration was given to the impression created, not only in what was said, using terminology they were familiar with during the interviews, but also through non-verbal communication: 'Looks, body postures, long silences, the way one dresses – all are significant in the interactional interview' (Fontana and Frey, 1998, p.68).

The combination of structure and flexibility allowed productive lines of enquiry to be followed up with appropriate probes, while questions that received little response could be discarded. However, an apparent lack of response to questions needed to be handled with care (Goetz and le Compte, 1984), for the interviewee might be trying to evade an issue which reveals important information for the study. An example of that from this study is when Victor, one of the trainees from the 2005-08 cohort, was asked about the extent to which the choice of examples is important in mathematics lessons, a key question in relation to the study. He responded with: 'To the children, obviously... very important, because, I mean... bit of a tricky one there... I don't know... I know what you mean, erm... I'm trying to explain it... erm...'. Victor seemed to be at a loss as to how to answer the question, perhaps suggesting he had little understanding of what it meant.

Throughout the interviews, assumptions made by the participants could be uncovered by encouraging them to give further detail to answers they had given. Participants' anxieties and fears, expressed through tone of voice or what was said, were listened for. For example, Naomi from the 2005-08 cohort was asked about why she might choose examples from a textbook:

> R: You said you used the books for the children sometimes, so if there's a page of examples in the book, do you look at those to decide which questions to use?
> S: Yes, I did that quite a lot actually, in some of my planning I'd put maybe question 1 to 12 and then maybe 14 to 18 or something, so they miss out a few that maybe were either too easy for them or a harder concept they didn't understand.
> R: So again, it's on ability?
> S: Yeah.
> R: Do you ever ask the children to make up their own examples?
> S: Yes.
> R: What sort of situation might you do it in?

S: We were doing problem solving and, especially with the higher attainers, if they finished their work, which they usually did very quickly, like they could finish a whole page of work in ten minutes if they understood the concept, I was getting them to think up questions of their own, I mean their own problem question...

Naomi's hesitancy to respond to this line of questioning suggests she may have been reluctant to discuss how she chose examples, maybe thinking her classroom practice was wrong in some way.

Active listening was practised during the interviews; this was reflected in the questions asked in response to the replies given by the trainee teachers. It was important to develop trust and confidence by supporting and encouraging the participants through verbal affirmation, nods, smiles and maintaining eye contact, responding positively to what they shared even when not agreeing with what they had said.

Another important strategy in the development of trust and rapport with the subject was to share aspects of the interviewer's experiences. Neuman (2000, p.370) suggests that:

> A field interview involves a mutual sharing of experiences. A researcher might share his or her own background to build trust and encourage the information to open up.

Whilst interviewing Suzy from the 2005-8 cohort, she commented on how teachers do not seem to be aware of theoretical work in mathematics education, and it was possible to give an insight into this research and potential future research as a way of responding to her remark and then lead on to the next question:

S: A lot of teachers don't know about these people because no one comes in and no one tells you about them and tells you who to read about.
R: That's interesting because what I'm looking at is how examples affect people who are on a course like this, and I'm wondering about how it's different to what practising teachers think. Let's talk about subject knowledge…how do you feel about your level of subject knowledge?

### 3.6 Selection of trainees for case studies

This section sets out the reasoning behind the choice of trainees for the case study analysis. The starting point was that some trainees had provided lesson plans but were not available for interview, whilst others offered to be interviewed but did not produce lesson plans. This led to a subset of each cohort from which to draw the sample, being those who produced lessons and were interviewed. For each cohort, a table of relevant data was compiled which included pre-course data, interview and diagnostic test results and module assignment grades (Table 3).

## Table 3: Sample group data ahead of selecting seven case study trainees

| Ref. No. | Pseud. | Interview length (minutes) | School Year Group | Pre-course data (GCSE, A-level) | Interview test score | DNT score | Y1 | Y2 | Y3 | No. of Lesson plans given for use | Grade for case choices |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ED01 | Anthea | | Y3 | B | 30 | 48 | 46 | 48 | 57 | 8 | B |
| ED02 | Sonia | 13 | Y2 | C | 55 | n/a | 51 | 58 | 62 | 15 | B |
| ED03 | Valerie | | Rec | C | 45 | 56 | 51 | 37 | n/a | 17 | C |
| ED04 | Michelle | | R/Y1 | C | 40 | n/a | 53 | 54 | 62 | 8 | B |
| ED05 | Joanna | 13 | Y5 | C | 35 | 44 | 51 | 52 | 52 | 14 | B |
| ED06 | Suzy | 12 | Y4/5 | A,B | 70 | 66 | 62 | 68 | 78 | 25 | A |
| ED07 | Colin | | Y6 | A | 100 | 73 | 56 | 74 | 68 | 24 | A |
| ED08 | Livvy | | R/Y1 | C | 35 | n/a | 52 | 55 | 55 | 3 | B |
| ED09 | Jane | 18 | Y4 | A,C | 40 | n/a | 50 | 58 | 55 | 24 | B |
| ED10 | Colette | 15 | Y3/4 | C | 65 | 49 | 52 | 48 | 36 | 30 | C |
| ED11 | Kirsty | | YR | B | 30 | 50 | 59 | 48 | 66 | 12 | B |
| ED12 | Kerry | | Y1/2 | C | 60 | n/a | 41 | 62 | 48 | 10 | C |
| ED13 | Dawn | 15 | Y4 | B | 50 | 52 | 65 | 57 | 80 | 22 | B |
| ED14 | Penny | 9 | Y2/3 | B | 50 | 65 | 60 | 72 | 75 | 18 | B |
| ED15 | Wendy | | Y5 | B | 80 | n/a | 54 | 72 | 62 | 30 | B |
| ED16 | Pam | | Y3/4 | A | 100 | 68 | 63 | 52 | 80 | 25 | A |
| ED17 | Lee | | Y3/4 | C | 55 | n/a | 54 | 48 | 67 | 24 | B |
| ED18 | Sharon | 23 | Y4 | A*,A | 90 | 71 | 71 | 67 | 68 | 33 | A |
| ED19 | Hayley | | Y3 | B | 60 | n/a | 40 | 61 | 52 | 25 | B |
| ED20 | Tania | | R/Y1 | C | 45 | n/a | 44 | 58 | 48 | 30 | B |
| ED21 | Victor | 18 | Y3/4 | C | 15 | n/a | 53 | 55 | 48 | 8 | C |
| ED22 | Kris | 14 | Y2 | B | 85 | 63 | 56 | 58 | 48 | 21 | B |
| ED23 | Judith | 34 | Y3/4 | C | 31 | n/a | 52 | 58 | 64 | 4 | C |
| ED24 | Naomi | 45 | Y4 | C | n/a | n/a | 40 | 45 | 40 | 21 | C |
| ED25 | Elizabeth | 36 | Y3 | n/a | n/a | n/a | 49 | 58 | 65 | 18 | C |
| ED26 | Rachael | 32 | Y3 | B | 71 | n/a | 60 | 58 | 65 | 19 | B |
| ED27 | Dawn | 46 | Y3 | C,E | 86 | n/a | 64 | 78 | 76 | 16 | A |
| ED28 | Helen | 50 | Y4 | B | 53 | n/a | 51 | 82 | 42 | 5 | B |
| ED29 | Clare | 39 | Y3/4 | A | 80 | n/a | 47 | 62 | 54 | 30 | B |
| ED30 | Sara | 18 | n/a | B | 78 | n/a | 55 | 66 | 54 | 13 | B |
| ED31 | Lauretta | 22 | Y1/2 | A | 74 | n/a | 50 | 48 | 52 | 20 | C |
| ED32 | Phillippa | 32 | Y3/4 | C | 75 | n/a | 66 | 62 | 62 | 24 | B |
| ED33 | Carl | 27 | Y1 | C | n/a | n/a | 40 | 48 | 52 | 12 | C |
| ED34 | Tracy | 24 | Y3/4 | n/a | n/a | n/a | 45 | 54 | 40 | 35 | C |
| ED35 | Ellie | | Y6 | C | 65 | n/a | 48 | 40 | 45 | 3 | C |
| ED36 | Mandy | 29 | Y3/4 | B | 89 | n/a | 63 | 78 | 72 | 37 | A |
| ED37 | Andy | 24 | Y5 | B | 42 | n/a | 48 | 57 | 62 | 20 | B |
| ED38 | Anna | 31 | Y3 | A | 89 | n/a | 57 | 68 | 67 | 17 | B |
| ED39 | Jane | | Y3 | C | 83 | n/a | 54 | 52 | 57 | 16 | C |
| ED40 | Lianne | | Y2 | C | 31 | n/a | 40 | 44 | 64 | 27 | C |

By considering the range of data for each trainee, it was decided to give each trainee an overall grade in the range A, B, C to indicate whether they were likely to be of higher attainment, middle attainment or lower attainment relative to the cohort. This grading was largely the researcher's subjective decision and was not arrived at by trying to calculate an overall result by any formulaic process, but sought to allow the selection of the case study students to form a purposive sample which, as fairly as possible, represented the range of students from two cohorts in terms of their past achievements, which informed my assessment of their potential for teaching mathematics.

Scoring highly on interview test, diagnostic numeracy test or modules assessments, and having studied mathematics to A-level, or achieving grade A at GCSE would all tend to suggest an A grade for this study, middling marks in each area might suggest a B grade, while a low pass in any category would be considered a C grade. A 'best fit' grading was then applied by looking across the range of data for each student, and then the seven case study students were drawn from all three grade bands to form a representative group.

Although the boundaries between the A/B/C categories would not be very distinct, the process of allocating the trainees to a particular category should be quite robust in identifying high, average and low attaining trainees in mathematical subject knowledge as related to teaching, and thus an effective process for selecting trainees to be part of the sample group for interviewing and subsequent selection, or otherwise, for the case study group.

The decision to choose the trainees to be the seven case studies was therefore then taken based on the following criteria:

- Year 3 trainee in either of the 2005-08 or 2006-09 cohorts
- Completing final school placement in the spring term of Year 3
- School placement was in Key Stage 2 and one of the most common school year groups
- Willing to submit mathematics lesson plans from final placement
- Mathematics topics taught were amongst the most common represented in the collection of lesson plans
- Willing to be interviewed about their planning and examples
- A selection of trainees from each of the A-C overall grading
- Interviews produced good quality data in sufficient depth for analysis

From the data and the criteria, seven trainees were selected for analysis of interview responses and lesson plans, against the research questions and literature themes.

### 3.7 Ethical considerations

Research subjects have the right to be informed about the nature and consequences of any studies in which they are involved, (Denzin and Lincoln, 2005). Voluntary informed consent was used for all potential research participants, including the provision of an information sheet outlining the aims, rationale, research questions and potential outcomes of the research. Participants' free choice to withdraw from the research process at any point was emphasised and participants were assured of and given anonymity and confidentiality in the research process. They were also given the opportunity to read and comment upon interview transcripts and any formally written material in order to guard against misrepresentation. Data was stored securely and used only for the purposes of the research and all data destroyed after use to ensure privacy and confidentiality for each subject.

# Chapter 4 Analysis and Discussion

## 4.1.1 Introduction

In this chapter the data collected from trainee primary teachers will be described, analysed and discussed against the research questions set out at the end of Chapter 2. Using qualitative data analysis the findings which emerged from the data will be outlined and these will be supported with evidence from the data sources including interviews and lesson plans from a number of case studies, selected from two cohorts of trainees at one United Kingdom university. The evidence from lesson plans is drawn from lessons that focused on aspects of teaching and learning number, since these were the most prevalent amongst the collected set of lesson plans. Clearly, there is scope for further work on lessons in the areas of shape and space, measure and data handling, but these are outside the scope of this study.

Key themes were identified that had emerged from the literature as being significant in relation to this study. These were:

- Planning procedures for teaching mathematics
- Resources that were used in planning and teaching
- Relationship between choice of examples and children's learning
- Consideration of examples in published materials
- Factors involved in the choice of examples
- Relationship between subject knowledge and choice of examples

### 4.1.2 Main findings from the study

The findings from the study are given briefly here, in relation to the research questions, and will be developed in detail through this chapter. With regard to the first research question 'What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?', the findings suggest that trainees use a variety of approaches, these being broadly summed up as 'reliant on the Primary Strategy', 'reliant on other sources' and 'use of their own knowledge'.

The second research question was 'How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?' This resulted in finding generally that primary trainees from the study are mostly able to recall aspects of theory from various module assignments during their course, but were collectively rather inconsistent in considering any theoretical frameworks specifically when planning to teach mathematics.

The third question asked 'Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?' In response to this question, each of the trainees identified with the idea that subject knowledge is related to choice of examples. However, their views on the range and scope of subject knowledge and their understanding of mathematical 'examples' led to a blurred interpretation of the relationship between subject knowledge and examples.

The higher attainers, as defined in the following sections, demonstrated greater understanding of the 'knowledge – examples' relationship and believed in each case that they chose better examples but also pitched their lessons at a level too difficult for pupils. The middle attainers had varying mathematical competence but firmly believed that better subject knowledge led to better examples being chosen. The two lower attainers had different views; one lacked confidence and needed more help with subject knowledge and choosing examples, which he did not regard as being related, whilst the other was confident but recognized she was still learning and felt that better knowledge of the pupils helped her choose better examples.

Amongst all the trainees, there was ample evidence in both interview discussions and from examples on lesson plans that all the case study trainees, regardless of attainment, used differentiation as a key feature of their examples. There were many instances of trainees talking about starting off with easier examples, so that every pupil could do them, and then making the examples progressively harder in order to challenge pupils of middle and higher attainment. This finding was not anticipated from the research questions but was drawn directly from the collected evidence and emerged during the analysis phase.

## 4.2 Case Study Trainees – Pen Portraits

### Case 1: Sharon

Sharon started her initial teacher education course immediately after leaving school with grade A at both GCSE and A-level mathematics, making her one of the highest attaining trainees in mathematics and in the most commonly represented demographic groups, namely females in the 18-25 year old category. In the university's interview test for mathematics, Sharon scored 90% which represents 9 correct answers on the 10-item test. She incorrectly answered a question involving a calculation of a total price when a given percentage of a number of identical items are sold at normal price and the remainder at half price.

After joining the course, Sharon completed the university's diagnostic numeracy test and scored 71 out of 77, giving an overall mark of 92%, the second highest in the cohort. Examination of her scores in each section of the test shows that Sharon lost 2 marks out of 11 available in the section on charts, tables and graphs, and a further 4 marks out of 11 were lost in the section on percentage and ratio.

It is worth noting at this stage that perhaps Sharon finds percentages difficult, having lost marks on both tests in questions on that topic. As the course progressed, Sharon achieved marks of 71, 67 and 68% in each of the Year 1, 2 and 3 course modules respectively (as described in section 3.4.2).

This evidence suggests that she is a capable student whose knowledge and ability in mathematics was applied well to her module assessments, demonstrating that she was able to relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments, although there is insufficient data to verify this speculation.

From examining all the pre-course data and results from the duration of the course, it was decided to consider Sharon as Grade A for the purpose of identifying different attainments across the case study trainees.

Sharon taught a Year 4 class for her final teaching placement and provided a total of 33 lesson plans from mathematics lessons. These covered the topics of subtraction, multiplication, angles, time and compass directions. For the purpose of this study the eight lessons on subtraction and multiplication were analysed to help give a consistent approach to the topic of number across each of the cases.

**Case 2: Suzy**

Suzy started her initial teacher education course immediately after leaving school with grade A at GCSE mathematics and grade B at AS-level, making her amongst the highest attaining trainees in mathematics and in the most commonly represented demographic groups, namely females in the 18-25 year old category. In the university's interview test for mathematics, Suzy scored 70% the 10-item test, although data was not available to determine which questions she answered correctly.

After joining the course, Suzy completed the university's diagnostic numeracy test and scored 66 out of 77, giving an overall mark of 86%, one of the highest in the cohort. Examination of her scores in each section of the test shows that Suzy lost just one mark out of 11 available in five of the seven sections, but dropped 3 marks out of 11 on charts, tables and graphs, and a further 3 marks out of 11 were lost in the section on significant figures and indices. As the course progressed, Suzy achieved marks of 62, 68 and 78% in each of the Year 1, 2 and 3 modules respectively.

This evidence suggests she is a capable student whose knowledge and attainment in mathematics were applied well to her module assessments, demonstrating that she was able to relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments. From examining all the pre-course data and results from the duration of the course, it was decided to consider Suzy as Grade A for the purpose of identifying different attainments across the case study trainees.

Suzy taught a Year 5 class for her final teaching placement and provided a total of 25 lesson plans from mathematics lessons. These covered the topics of time, multiplication, decimals, division, number sequences, and fractions. For the purpose of this study each of the lessons on multiplication and fractions are analysed to give a consistent approach to the topic of number across each of the cases.

**Case 3: Dawn**

Dawn started her initial teacher education course after a non-teaching career and the bringing up of a family, achieving grade B at GCSE mathematics. Dawn is amongst a middle attaining group of trainees in mathematics and in a less commonly represented demographic group, namely females in the over-25 year old category. In the university's interview test for mathematics, Dawn scored 50% on the 10-item test, answering a range of questions correctly but also being unable to answer some questions. After joining the course, Dawn completed the university's diagnostic numeracy test and scored 52%, although detailed data on which questions were answered correctly was not available. During the course, Dawn achieved marks of 65, 57 and 80% in each of the Year 1, 2 and 3 modules respectively.

This evidence indicates she is a moderately capable student whose knowledge and attainment in mathematics were applied satisfactorily to her module assessments, demonstrating that she was able to relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments, although her high mark in Year 3 might suggest that Dawn applied her earlier learning well to become a higher attainer towards the end of her course. From examining all the pre-course data and results from the duration of the course, it was decided to consider Dawn as Grade B for the purpose of identifying different attainments across the case study trainees.

Dawn taught a Year 4 class for her final teaching placement and provided a total of 22 lesson plans from mathematics lessons. These covered the topics of fractions, 2D and 3D shape, symmetry, angle, perimeter and area. For this study, lessons on fractions are examined to be consistent in analysing number across the cases.

**Case 4: Rachael**

Rachael started her initial teacher education course immediately after leaving school with grade B at GCSE mathematics, making her amongst the middle attaining trainees in mathematics and in the most commonly represented demographic groups, namely females in the 18-25 year old category. In the university's interview test for mathematics, Rachael scored 71%. As the course progressed, Rachael achieved marks of 60, 58 and 65% in each of the Year 1, 2 and 3 modules respectively. It is speculated that these figures indicate she is a moderately capable student whose knowledge and ability in mathematics were applied well to her module assessments, demonstrating that she was able to relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments. However, there is insufficient data to verify this speculation and it would be necessary to triangulate this evidence with, for example, classroom observation data or a written test of mathematical knowledge for teaching. From examining all the pre-course data and results from the duration of the course, it was decided to consider Rachael as Grade B for the purpose of identifying different attainments across the case study trainees.

Rachael taught a Year 3 class for her final teaching placement and provided a total of 19 lesson plans from mathematics lessons. These covered the topics of 3D shape, capacity, data handling, addition, subtraction and multiplication. For the purpose of this study the lessons on addition and subtraction were analysed to give a consistent approach to the topic of number across each of the cases.

**Case 5: Andy**

Andy started his initial teacher education course immediately after leaving school with grade B at GCSE mathematics, making him amongst the middle attaining trainees in mathematics but in a poorly represented demographic group, namely males in the 18-25 year old category. In the university's mathematics interview test, Andy scored 42% which meant he was only just considered eligible to get a place on the course. As the course progressed, Andy achieved marks of 48, 57 and 62% in each of the Year 1, 2 and 3 modules respectively, which suggests steady progress in his academic development as a prospective primary teacher.

This evidence indicates he is a student whose knowledge and ability in mathematics were applied well to his module assessments, demonstrating that he found he could relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments, although there is insufficient data to verify this speculation. From examining all the pre-course data and results from the duration of the course, it was decided to consider Andy as Grade B for the purpose of identifying different attainments across the case study trainees.

Andy taught a Year 5 class for his final teaching placement and provided a total of 20 lesson plans from mathematics lessons. These covered the topics of coordinates, translation, 2D shape, area and perimeter, time, addition and subtraction, decimals, fractions and division. For the purpose of this study the lessons on addition, subtraction, fractions and division were analysed to give a consistent approach to the topic of number across each of the cases.

**Case 6: Victor**

Victor also started initial teacher education immediately after leaving school, although he had performed less well in school mathematics, achieving with grade C at GCSE. As a male, Victor represents a minority for primary trainees, in a poorly represented demographic group, namely males in the 18-25 year old category. In the university's interview test for mathematics, Victor only scored 15% at the first attempt, which represents just three questions on the 10-item test where he was awarded half a mark.

He was only able to score marks on questions involving calculation of a total price when a given percentage of a number of identical items are sold at normal price and the remainder at half price. Before being able to join the course, Victor re-took the interview test and on the second attempt at the same test he scored 55%, this time gaining a full mark on five of the questions and half a mark on one question. Of the three questions which gave Victor half marks at the first attempt, one was fully correct at the second attempt, one again scored half a mark, but on one question he failed to score on the second attempt.

Once he had started the course, Victor completed the university's diagnostic numeracy test but no data was available on his result. However, as one of the lowest scoring trainees he was invited to take the additional mathematics module 'Confidence Counts' which covers a range of GCSE level topics to help improve trainees' mathematical subject knowledge. At the end of the module the trainees sit an exam and Victor scored 54%. During the 3-year course, Victor achieved marks of 53, 55 and 48% in each of the Year 1, 2 and 3 modules respectively.

These results suggest he is a student whose knowledge and attainment in mathematics did not appear to be strong in the module assessments, demonstrating that he had a continuing issue with subject knowledge. However, in his school placements, Victor showed he was able to relate subject knowledge to pedagogic knowledge through his high grades during the placements. From examining all the pre-course data and results from the duration of the course, it was decided to consider Victor as Grade C for the purpose of identifying different attainments across the case study trainees.

Victor taught a Year 3/4 class for his second year and final teaching placements and provided a total of 8 detailed plans from mathematics lessons. These covered the topics of 3D shape, units of time, data handling, addition, subtraction and multiplication. For this study the three lessons on addition, subtraction and multiplication are analysed.

**Case 7: Naomi**

Naomi started her initial teacher education course immediately after leaving school with grade C at GCSE mathematics, making her amongst the lower attaining trainees in mathematics and in the most commonly represented demographic groups, namely females in the 18-25 year old category. As the course progressed, Naomi achieved marks of 40, 45 and 40% in each of the Year 1, 2 and 3 modules respectively. These figures indicate she is a student whose knowledge and attainment in mathematics were not applied well to her module assessments, demonstrating that she found it difficult to relate the subject knowledge to the pedagogic aspects which were assessed in the module assignments, although there is insufficient data to verify this speculation.

Further evidence to support this hypothesis could come from close analysis of Naomi's lesson planning and her teaching, although the latter was not possible in this study. From examining all the pre-course data and results from the duration of the course, it was decided to consider Naomi as Grade C for the purpose of identifying different attainments across the case study trainees.

Naomi taught a Year 4 class for her final teaching placement and provided a total of 21 lesson plans from mathematics lessons. These covered the topics of addition and subtraction, multiplication and division, function machines, rounding and estimation, money and negative numbers. For the purpose of this study the lessons on addition, subtraction, multiplication and division were analysed to give a consistent approach to the topic of number across each of the cases.

### 4.3 Dimensions of the Knowledge Quartet

Consideration is now given to how the case study trainees' interview responses and lesson plans relate to current theoretical frameworks. For the purpose of this study, the theory is drawn from two particular models which were outlined in the review of literature in Chapter 2. The first is the framework of Rowland *et al.* (2009) which arose after examining videotapes of trainees and identifying features of the lessons. The second theoretical framework being used here is described by Watson and Mason (2005) in their work which considers examples from a number of perspectives, again as described and reviewed in Chapter 2. In Rowland *et al.'s* study, eighteen categories were identified which were later organised into four main groups, the whole framework being given the name 'The Knowledge Quartet'.

It should first of all be stated that none of the trainees in either cohort who were interviewed had heard of the Knowledge Quartet. This was partly because the framework was not included as part of students' required reading for the B.Ed programme in general and the mathematics modules in particular, and also none of the trainees had come across the framework in their own reading and research during the course. In some regards this is unfortunate, however, discussions about future content of the modules is beyond the scope of this study. Analysis of the case study trainees' lesson plans and interviews revealed that despite not being aware of the Knowledge Quartet, there were elements of the framework in their planning and interview transcripts and these will be considered now.

### 4.3.1 Foundation

During the development of the Knowledge Quartet, a number of codes based on video-taped lessons were identified as part of the grounded theory approach taken by Rowland *et al.* (2009). Two of the contributory codes which went towards the formation of the Foundation dimension were 'overt subject knowledge' and 'adherence to textbook'. In this section, the lesson plans and interview data from the seven case study trainees is examined for evidence of the extent to which they use or demonstrate these aspects in their planning and teaching.

#### 4.3.1a 'Overt subject knowledge'

Starting with the more able trainees, both Suzy and Sharon showed that they were aware of the need to ensure that their subject knowledge was at a suitable level to teach the topics they were allocated on their placements.

Overt subject knowledge is one of the factors identified within the Foundation dimension of the Quartet and not only did the trainees know that they needed good subject knowledge, they also recognized the need to improve if it was not sufficiently good for a topic.

As Suzy recalls: 'I'd go into the maths and have to practise it to make sure I know what to do, 'cause they all change', and Sharon identified the need to refer to guidance on what to teach, stating: 'with the NNS it sort of like, breaks it down more like for terms, it's just the objectives'.

Dawn identified in her interview that her subject knowledge may be outdated as she is a mature student and felt that things had changed since she was at school. For example, she describes one aspect of teaching subtraction which she regards as different nowadays:

> First of all I would gen up on the topic because it's many years since I did maths as a child, erm... and some things I find hard are taught differently, for example, the way you can't borrow one and pay back anymore, so... I have to double check the way things are done now.

One of the trainees identified as less able in mathematics is Victor, whose interview revealed not only that he had identified weaknesses in his level of mathematics, but also that he used this as a reason to adapt his teaching in a way which might be thought of as very worrying:

> 'I always struggled with maths to be honest... I think sometimes it can be limiting, 'cause sometimes you look at things and think I don't quite understand that fully, so I won't bother teaching... you know, I might leave that today'.

#### 4.3.1b 'Adherence to textbook'

In the findings at the start of this chapter, it was identified that some trainees tend to rely on the PNS to ensure they have covered the relevant objectives and planned appropriate activities. However, in terms of using PNS materials or published schemes, all of the case study trainees showed evidence that they rarely change the examples they find. Most trainees prefer to 'adhere to the textbook', a facet of the Foundation dimension of the Knowledge Quartet from Rowland *et al.* (2009). It might be expected that less able trainees would be more likely to follow the textbook closely if their own level of subject knowledge is weak.

Looking firstly then at Victor and Naomi from the case study group, Victor reported that his plans are: '...always from some kind of framework, that the teacher has given us', implying that the plans are 'ready-made' for use and cannot or need not be changed. Naomi, whose school used the Hamilton Trust plans, described how easy they seemed in terms of use for planning, as they: '...used the Hamilton Trust website, and it was easy for them just to go and print and it comes off... with the average Year 4 work, then I could pitch it to different groups'.

In the middle attaining group of trainees, Andy talked about using the materials from the Abacus scheme and: '...as long as they meet the criteria and objectives that you're trying to teach, I think that's the main key'. Rachael noted how teachers in her placement school were actually defying school guidance and:

> Quite a few of the teachers were using a scheme they weren't supposed to, and they'd take the worksheets from it because it literally just told you to do task 1 and task 2, like that.

Dawn, the mature trainee, expressed surprise when she recalled how one teacher used the textbook in her placement school, saying she was a:

> ...very experienced teacher but actually taught the maths with the scheme book open on the desk in front of her, and taught exactly from the book, which surprised me because she said she'd been teaching for more than 20 years and yet she obviously still felt the need to do that.

From the combined responses of the middle attaining group of trainees, it would seem that they feel there should not be an adherence to the text, but if there was, it was either to ensure coverage of PNS objectives or because the teachers in the school were not confident of their subject knowledge and relied on routine methods which had served them well over a number of years.

Just as it might be expected that less confident trainees might rely on textbooks to counter their lack of deep subject knowledge, then possibly the more able trainees should be able to plan lessons and choose examples without adhering to textbooks so much. However, the interview transcripts suggest that following the textbook is still evident in this group of trainees. This could be due to the fact that although they are the more able trainees in terms of mathematical attainment, they are equally inexperienced in teaching and planning mathematics and so may be relying on schemes which are written to help teachers in having lessons and examples already set out. Suzy refers to having to: 'check the text books they are supposed to be using' and Sharon reports that the teachers in her school were: '...looking for a scheme of work and things, because they said it's really hard to know what to teach when'.

It is important to note, after considering trainees' approaches to planning and choice of examples, that each trainee will be constrained to some extent or other by each school's approach and policy to planning in mathematics and how this impacts on any placement trainees that teach in the school. In some cases the PNS may have been followed extremely closely whereas in other schools the non-statutory guidance of the PNS may have been used as support alongside other approaches, such as published schemes.

Some trainees gave a better indication than others in this respect, for example, Suzy speaks about her school making a concerted effort to adopt the non-statutory guidance: 'They were trying to use the national... the new strategy', but also mentions that she needed to: 'check the text books they are supposed to be using'.

Sharon also described how her school: '…started using the PNS, but they didn't have any, sort of, strategies in place as such'. She highlights the situation many schools may have found themselves in over the past few years, trying to incorporate the latest version of the PNS (which at the time of writing has effectively been abolished from 2011 ahead of the new Primary Curriculum which is due to come into operation from September 2013), but still trying to find published schemes of materials for lesson ideas and children's activities.

Looking at the transcripts from the other case study trainees, Rachael speaks about her school using the: 'Primary Framework, the new Strategy, and just check on the strands and blocks', but notes that the school did not have a scheme in place. Andy's school appeared to be combining both the Strategy and the Abacus scheme:

> They used the renewed framework online, however they mixed it with Abacus, so they still used Abacus but then they referred back to the framework just to make sure that they are meeting the criteria and the objectives.

This suggests the school was satisfied with the content of the scheme but were not entirely trusting that it matched the requirements of the PNS. This may not have been particularly helpful to the trainees. In Dawn's case, the school seemed to use the scheme to ensure the PNS was being covered:

> Their maths co-ordinator sort of said, 'oh well, we're not meeting this… sort of, certain targets', and she was trying to get the teachers to go back to using the Abacus scheme more logically than they had been doing.

This may have given Dawn the idea that by following the scheme, the objectives of the PNS will be met, whereas Andy's experience was that following the scheme did not guarantee that the PNS objectives were all being met.

Victor also found that his placement school were using the Abacus scheme but he felt they needed to check with the PNS to ensure appropriate coverage of objectives: 'We used Abacus Evolve, and it was the brand new one, and they pretty much do it all for you', and he later added: We did so much from Abacus, it was just a case of checking... with what was supposed to be going on with the PNS'.

The only trainee amongst those selected as case studies that had a different experience in terms of using PNS guidance was Naomi, whose placement school relied on the planning guidance from the Hamilton Trust. She described using plans from the Hamilton Trust as her main source, but was prepared to look at the PNS for further help:

> I was given plans from the Hamilton Trust and I adapted those into my own planning to suit the class, but if I wasn't given those then I would probably use the Primary Framework, probably the 'I can' statement or something like that and the suggested tasks they have, because all you need is something little to get your brain going.

Considering all the evidence about trainees' approach to planning and choosing examples, it would reasonable to suggest that only the trainees who were identified as higher attainers gave the impression of being confident enough in their planning and teaching to not have to rely on the PNS and could plan using other sources.

With each of the other case study trainees, their interviews and transcripts gave the impression that they always reverted to the PNS to help them with planning, either because the school used it or because they believed it to be the 'right' way, an impression gained from their training or the school setting, or both.

In each case, there is clear evidence from the lesson plans and interviews that in terms of planning mathematics, the trainees all demonstrate a tendency to 'adhere to the textbook', a feature of the Foundation dimension of the Knowledge Quartet of Rowland *et al.* (2009).

Before moving on from the Foundation dimension of the Knowledge Quartet, there was some evidence of trainees using other aspects in that dimension of the theoretical framework which will be discussed briefly here. Sharon, for instance, in raising with the class teacher the question of which topics she needed to cover on her placement, showed an 'awareness of purpose' in her planning in terms of the subject knowledge needed for each topic. She was also aware of relevant theoretical underpinning in her teaching, referring to books she had used for her undergraduate dissertation and making direct reference to theory. Rachael referred to an assignment she had written as part of her professional studies in which she:

> ...did all about the 6 Rs for mental and oral starters... they stand for different things, there are rehearse, recall...there are six different ones from the Primary Framework and so now I realize that when I'm doing it, I think this is a rehearse or a recall, so I'm using it without realising.

Rachael also makes reference to children's learning preferences, describing how she tries to make use of visual, auditory and kinaesthetic approaches when planning mathematics, as well as using elements of the ELPS model referred to in Liebeck (1990). Both of these approaches are also mentioned by Naomi, although she labels children as 'visual learners' rather than talk about them having a preference for learning visually. However, all of these trainees referred in some way to being aware of theoretical models that influence their planning, and the likely source of this awareness is the training they receive on the B.Ed course.

### 4.3.2 Transformation

The second dimension of the Knowledge Quartet is that of Transformation and this contains three distinct contributory codes from the original data collection:

- Choice of examples
- Choice of representation
- Demonstration

It is the first of these aspects, 'choice of examples', which is the focus of this study and so some analysis of how this contributory code is represented in the case study data is now set out.

Before identifying particular evidence from the trainees, it should be pointed out that the trainees were all made aware that the study was about their choice of mathematical examples and so any mention of this in their interview was not random but a response to particular questions about that aspect of their planning. The fact they referred to choice of examples on a number of occasions is therefore significant, however, the quality of their responses in regard to this element of the Knowledge Quartet gives good indication of their approach to choosing their examples. Each of the trainees for the case studies provided a selection of lesson plans from their final school placement and these were analysed for evidence of how the trainees selected their examples for teaching, as well as the examples they chose for the children to work on. The examples that appeared in the lesson plan evidence will now be described and commented on in relation to the theoretical framework of the Knowledge Quartet (Rowland *et al.*, 2009) and the work of Watson and Mason (2005) on examples, and it was decided that the best way to organise this evidence is by considering each attainment group of trainees, rather than take each facet of the dimension in turn.

#### 4.3.2.1a Higher attaining trainees – Suzy

Suzy provided a series of lesson plans in which she was teaching her Year 5 class to multiply using a 'grid and column' method. In the first lesson, Suzy begins with a series of mental calculations involving the multiplication of a single digit by a multiple of 10, the first example being 6 x 40. There is no significant issue with the choice of numbers in this example, but Suzy's chosen procedure for explaining how to calculate the result is rather unfortunate. The following note on the lesson plan suggests that Suzy will ask the children to calculate 6 x 4 and then multiply the result by 10. She notes to herself in a bracket '(add a zero)', suggesting that she will reinforce the misconception that many children and adults have, namely that adding a zero is the same as multiplying by ten.

The lesson develops to include multiplying a single digit by a multiple of 100, with 3 x 400 offered as a first example of this type. The numbers are not an issue, just as previously, but the lesson plan notes this time that the calculation is equivalent to 3 x 4 and 'add 2 zeros'. Once again the misconception regarding multiplication by powers of 10 appears and is reinforced by the trainee teacher. Suzy's first few examples from the worksheet in this lesson have been considered from the perspective of the sequence of examples Suzy chose during the planning process, and now other examples from Suzy's lessons are considered from the point of view of their identification with the Transformation dimension of the Knowledge Quartet.

The worksheet referred to contains a series of examples of multiplication using a grid method. The partial results formed do not necessarily have the same number of digits, caused by the choice of digits in the multiplicand and the multiplier. In the next section, the following lesson's worksheet is considered.

The third lesson plan gives little in the way of examples, but the accompanying worksheet offers a number of examples to discuss. It opens with a worked example, 34 x 26, using the method outlined in lesson 2, and then sets out a series of six word problems which require children to extract the appropriate multiplication problem and calculate the answer using the same method.

The first question on the worksheet is as follows: There are 40 crayons in a pot. How many crayons in 15 pots? This question provides a relatively simple calculation in multiplying a multiple of ten by a 2-digit number containing the digits 1 and 5 which are among the easier multiplication tables to calculate with. The grid and column method for this calculation then becomes:

**40 x 15**

| x | 40 | 0 |
|---|---|---|
| 10 | 400 | 0 |
| 5 | 200 | 0 |

| Th | H | T | U |
|---|---|---|---|
| | 4 | 0 | 0 |
| | 0 | 0 | 0 |
| | 2 | 0 | 0 |
| | 0 | 0 | 0 |
| | 6 | 0 | 0 |

40 x 15 = 600

With the whole multiple of ten ending in zero, the grid and column method produces two partial calculations of zero for the addition. Some children may be able to disregard this if they recall that multiplying by 40 is the same as multiplying by 4 and then by 10.

Or, as Suzy described in an earlier lesson, multiply by 10 and 'add a zero'. This example therefore introduces some difficulties regarding place value conventions which are sometimes overcome by colloquial phrases such as 'add a zero' which will fail to work with decimal numbers.

The second example from the worksheet is: One mug has 22 spots. How many spots have 34 mugs got? It could be argued that the context for this question is spurious, since mugs are not generally distinguished by the number of spots on their external pattern. However, from the point of view of the numbers to work with for the grid and column method, this example provides all non-zero digits, giving a set of four non-zero partial calculations in the working:

**22 x 34**

| x | 20 | 2 |
|---|---|---|
| 30 | 600 | 60 |
| 4 | 80 | 8 |

| Th | H | T | U |
|---|---|---|---|
| | 6 | 0 | 0 |
| | | 6 | 0 |
| | | 8 | 0 |
| | | | 8 |
| | 7 | 4 | 8 |

22 x 34 = 748

The third example is as follows: One banana costs 10p. How much do sixteen bananas cost? This appears to be a backward step by providing a much easier and more trivial example than the first two. Multiplication by ten has been introduced as a step along the way to using the grid and column method, making this example one that can readily be solved mentally. This can be done either by recognizing the column movement of digits when multiplied by ten or by using the 'add a zero' technique which Suzy has previously alluded to. The setting out of the calculation for this example has been omitted since it is trivial.

The fourth example is as follows: There are 45 trees in each of the 5 local schools. How many are there in all 5 schools? This example moves back to using non-zero digits, enabling the grid and column method to be used less trivially than in the previous example, but with only a units digit as one of the numbers in the product, this again represents a simpler example than those earlier on the sheet and the calculation is again omitted here.

The fifth example on Suzy's sheet (incorrectly numbered 6 on the original) is: There are 35 people in a running team. How many people in 17 teams? On examining the numbers used, this appears to be the most complex example yet encountered on the sheet. There are four non-zero digits used in the calculation and none of them are repeated. Further, the digits include a 7 which represents one of the more difficult multiplication tables. The grid and column calculation is as follows:

**35 x 17**

| x | 30 | 5 |
|---|---|---|
| 10 | 300 | 50 |
| 7 | 210 | 35 |

| Th | H | T | U |
|---|---|---|---|
| | 3 | 0 | 0 |
| | | 5 | 0 |
| | 2 | 1 | 0 |
| | | 3 | 5 |
| | 5 | 9 | 5 |

35 x 17 = 595

The final example on the worksheet is as follows: Apples cost 89p each. How much for 34 apples? The sheet ends with another example using four non-zero digits and also the digits 8 and 9 which are amongst the more difficult multiplication tables. The calculation is therefore the most complex on the sheet and perhaps appropriate for the final example.

The calculation is as follows:

| **89 x 34** | | | Th H T U |
|---|---|---|---|
| x | 80 | 9 | 2 4 0 0 |
| 30 | 2400 | 270 | 2 7 0 |
| 4 | 320 | 36 | 3 2 0 |
| | | | 3 6 |
| | | | 3 0 2 6 |

89 x 34 = 3026

This final example has a money context and so the final answer needs to be interpreted as a total amount, which is £30.26 rather than the 3026 which is the total in pence.

The teaching of grid multiplication, as discussed here from Suzy's lesson plans, resonates with findings by Rowland (2008) in which he describes a lesson by 'Laura', who offered pupils a series of multiplication problems involving multiplying a 2-digit number by a single digit. Laura's examples were intended to become progressively more challenging, however, it was remarked that many of her examples could have been solved using mental methods such as doubling. If there was an element of increasing challenge, then Laura should have been providing examples which required recall of products or examples where 'carrying' from units into tens or tens into hundreds was part of the calculation. Suzy's examples above seem to provide evidence of carrying in many cases, but she randomizes the level of challenge which may not have been helpful for the children.

Summing up the examples used by Suzy, she appears from the plans to have an approach to choosing examples which is to some extent quite random, with examples moving from simpler to more complex and back again with no apparent purpose. She had stated in her interview that she likes to; 'pick out certain questions for them to do rather than just work through', and then later she also said: 'I'd try to mix it up sometimes' and the evidence from the worksheet above seems to demonstrate that she does indeed mix her examples. In this case, it is important to note that:

> It is very common for learners to identify concepts with one or two early examples that they have been shown by the teacher. Because these early examples are often simple ones, the learner is left with an incomplete and restricted impression of the concept (Watson and Mason, 2005, p.66).

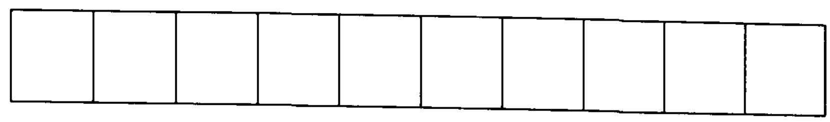
#### 4.3.2.1b Higher attaining trainees – Sharon

The other case study trainee identified as being more able mathematically, Sharon, was teaching a Year 4 class and for this analysis, her lessons on subtraction are considered. In the first lesson, the key vocabulary was the phrase 'take away', suggesting a model for subtraction which involved physically removing one number from a larger number and counting what remains. This was supported by the use of 'minus' which also represents the take away method of subtraction. There is mention on the lesson plan that Sharon will point out to the children that 'subtraction is the opposite/inverse of addition' which, whilst being correct, may lead to a mixed model being demonstrated if the examples selected are worked using a complementary addition approach, where the difference between the larger and smaller numbers in the calculation is determined by adding on from the smaller amount.

This type of calculation is characterized by use of a minuend and a subtrahend, the difference between these representing the answer to the calculation. As Rowland (2008) points out, each of these elements when choosing examples of this type of calculation can be a dimension of variation, although the choice of numbers for any two of these elements will naturally determine the value of the third.

Choosing different numbers for each of the three elements helps learners to discern the purpose of each of the minuend, subtrahend and difference within the calculation, although in a lesson analysed by Rowland, 'Naomi' (not the same Naomi as in this study) used as a first example of subtraction $4 - 2 = 2$. In this case the subtrahend and difference took the same value, thereby obscuring the role of the variables. With regard to Sharon's examples of taking a single digit from 10, this situation would occur with $10 - 5 = 5$.

In the first part of the lesson, since number bonds to 10 are being considered, 10 is the selected larger number in each subtraction problem. The first example listed is $10 - 1 = \square$, which might be considered a good starting example since it involves the smallest amount of taking away from the selected number. However, the choice of numbers in this example is not supported by an appropriate choice of method, as the children are asked to draw a block of ten squares and colour one square to represent the '*one*' to be taken away. From this coloured diagram, the children are to complete the number sentence as $10 - 1 = 9$ by counting the uncoloured squares on their diagram.



This example provides a straightforward calculation but does not model the 'take away' method accurately, since nothing is actually removed from the representation of 10. As far as can be determined from the lesson plan, the children then worked on some other similar problems, each time drawing a block of ten squares and colouring in a specified number, an exercise in partitioning, rather than 'take away'.

These problems would have been listed on the interactive whiteboard (IWB) and there is no record of the actual sequence of the problems. By examining the transcript of Sharon's interview, it is possible to gain insight into her approach when choosing examples for children to work on independently. When asked about the factors involved when choosing examples, Sharon responded with:

> I normally start off quite easy because it's better to start off with a really easy one that they can do, because they already get the confidence up that they can do it...if they can't do what you think is easy, 'cause you can sometimes pitch it too high, if you think that's easy they can still struggle with it, so then you can lower it or higher it as you need to.

This approach was also evident from Sharon's description of how she might choose examples specifically for a worksheet or a set of problems for the IWB:

> It often took quite a while to explain it, we would do a lot on whiteboards and then they'd have a sheet and that would be about it, but in my own planning I would always write out, so instead of using a worksheet I wrote the questions on the board sometimes, so then I could go to the children who'd finished and write questions in their book, and I would get more...progressively more difficult.

It might be surmised that Sharon's examples for the children continued in some sort of sequence, 10 – 1, 10 – 2, 10 – 3, and so on. Such a choice is certainly apparent in the extension work on the lesson plan, where the activity is carried out using number bonds to 20.

It is not evident whether the children are required to draw a line of 20 squares for each example and colour in the amount to be 'taken away', but the number sentences are required to be recorded in sequence:

$$20 - 1 = 19$$
$$20 - 2 = 18$$
$$20 - 3 = 17, \text{ and so on.}$$

Again there is no element of physically removing the smaller numbers, and so in this activity the choice of examples appears to lead to pattern formation rather than an understanding of the subtraction process, although this is opinion which is not supported by direct evidence. Continuation of this sequence of subtractions will lead the children eventually to the case 20 – 20 = 0 and it is likely that they will stop at that point. At this point, the child has encountered a boundary example, as it lies on the boundary between positive and, later to be discovered, negative numbers (Watson and Mason, 2005).

Finally for this lesson, examples from the number bonds to 10 are used in the plenary to demonstrate the inverse connection between subtraction and addition. Children are asked to consider 10 – 1 = 9, and 'walk backwards through the sum' to arrive at 9 + 1 = 10. This serves to reinforce the idea of the inverse.

Notwithstanding the use of 'sum' to describe a subtraction, there is a problem with these examples not enabling learners to relate the physical action of taking away and the inverse operation of combining two sets to produce an additive total. In summary, this lesson uses a range of numbers with which to carry out subtraction problems, but it is the choice of method rather than the choice of numerical examples which could be misleading to learners.

Sharon's next lesson follows on the next day after the one on number bonds to 10 and 20, so the increase in conceptual difficulty is surprising. Following an introduction which revises examples such as 10 – 1 = 9 from the previous day, Sharon's lesson plan directs her to 'use more difficult sums so the children can see the theory, for example, 99 – 20 = 79'.

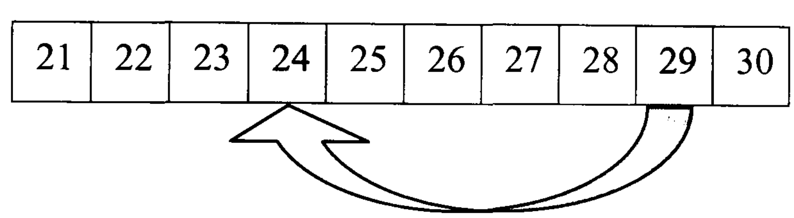
The method used in the first lesson involved identifying the difference, for example, between 10 and 1, by drawing 10 squares and colouring one, then counting what remained. By moving onto 2-digit subtraction, the efficiency of such a method is questionable since it is unlikely that children will be asked to draw a block of 99 squares and colour 20 before counting the 79 that remain. By using the hundred square, subtraction is perhaps best carried out by making use of the place value arrangement of the numbers, having 10 in each of ten rows. Subtraction of 10 can be seen as moving up one row, subtraction of 20 is moving up two rows and so on. The chosen example, 99 – 20 can then be easily seen by starting at 99 and moving upwards 2 rows to reach 79.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

The hundred square in this case acts as a calculation aid and whilst it produces the correct answer to the subtraction problem, it is by a method which fails to demonstrate the physical connection between taking 20 items from a group of 99 and the number sentence 99 – 20 = 79.

One interpretation of this situation could be that it uses a 'procedure that is not a sensible one' (Rowland *et al.*, 2009) although the discrepancy between the procedures is not with regards to efficiency but more in terms of modeling the process that is recorded by the number sentence. This issue of using the hundred square for addition and subtraction was described by Rowland *et al.* (2009) in discussing a lesson by 'Chloe'. She was attempting to use the hundred square to teach children to add 9, by looking at how the numbers are arranged and how adding 9 can be accomplished by moving back one place along a row and then down to the next row below. It was recognized that she was attempting to help children learn to verbalise the procedure for adding 9 on a hundred square, rather than learn a procedure, however, there was little evidence that she was helping children learn to round and adjust numbers.

In the main teaching part of the lesson, Sharon demonstrates how to subtract on the number square by either moving upwards to subtract tens or moving backwards along the rows to subtract units. Her first listed example is 30 – 10, which suggests she has chosen to focus on the subtraction of multiples of 10 first before introducing subtraction of units. It might be suggested that following on from number bonds to 10 in Lesson 1, subtraction of units on a hundred square would be a more suitable learning step. This then appears on the plan in the form of 29 – 5, which will involve a simple move of 5 places backwards along the row in which 29 appears, providing the number sentence 29 – 5 = 24.




There appears to be a deliberately considered choice here, with the 9 units in 29 being chosen to enable the subtraction to be worked along a single row of the hundred square without needing to 'bridge through a ten'. If the chosen example had been 23 – 5, then the procedure would have followed the sequence 23, 22, 21, 20, 19, 18. This would involve moving backwards along the row to 21, then jumping up a row and re-positioning at the end of the row at 20 before finishing the 'backward' count to 18.

This resonates closely with the example of Chloe from the work of Rowland *et al.* (2009). Again, at this point the children would have been completing examples from the IWB, but later the lesson plan indicates how the children will be shown how to partition 2-digit numbers in order to subtract them from larger numbers.

In this case, instead of counting backwards along rows for single digits or moving up rows to subtract tens, there is a combined method which deals with subtraction of 2-digit numbers which are not multiples of ten.

The first example given is a typical example of what Rowland *et al.* (2009) describe as one which obscures the role of the variables; here Sharon has chosen to demonstrate the new combined method with 50 – 25, which will give the solution 25, meaning that the subtrahend and the difference are the same number. This example has the possibility of causing confusion to some learners and could have been avoided, given that there are 35 possible numbers that can be subtracted from 50 and where the number has 2 digits and no zero in the units column.

The plenary for the second lesson moves the children to consider subtracting 2-digit numbers mentally, perhaps by visualising the hundred square. The listed examples for this on the plan give subtractions 32 – 10 and 31 – 20. It is clear that Sharon has chosen subtraction of multiples of 10 to make the mental calculation relatively easy, but looking at the problems there is a confusing juxtaposition of the digits 2 and 1, and given the similarity of the numbers chosen it is likely that some learners may decide the answer is the same in each case as the same digits are represented.

If Sharon's level of pedagogical content knowledge is sufficiently high, she may notice that the answers are linked, being 22 and 11 respectively, and that investigation of similar pairs will yield similar results. The juxtaposed digits offer the multiples of 11 that result in the answers.

For example, in 43 – 10 = 33 and 41 – 30 = 11, the juxtaposed digits 1 and 3 lead to answers 11 and 33. Sharon's final example seems to suggest an awareness of 11 but she fails to develop this link, simply listing 22 – 11 as her final subtraction for the children to solve mentally. This example again will lead to a subtrahend that is the same as the difference and so the example 'obscures the role of the variables' (Rowland *et al.*, 2009).

In summarising this lesson, Sharon appears to have extended the learning of subtraction in a considerable conceptual leap, moving from subtraction that can be readily worked with physical materials to using a resource, the hundred square, as a means of calculating easily. Her choices of examples included at least one which appeared to hide the role of the variable when there was a wide range she could have chosen from.

The third subtraction lesson from Sharon offers an immediate example of ambiguity in the objective, since it is not clear how the two given numbers are to be used. If the numbers are to be written in an addition problem, it is not clear whether they are the numbers to be added to give a total, or whether the larger number is intended to be the total. In subtraction, the numbers may form the minuend and subtrahend, or the smaller number could be intended as the difference. The lesson begins with a bingo-type activity involving rolling two dice and the numbers that show are to form a subtraction calculation. The children write down 3 numbers from the range 0 to 5 in advance, representing all possible results and if the subtraction gives one of their numbers they can cross it out.

The example Sharon lists on her plan is 6 – 3 = 3, another of those where the role of the variable may be perhaps obscured (Rowland *et al.*, 2009). However, there is a further issue with the range of possible answers, since these occur with different frequencies. An answer of 5 can only happen if 6 and 1 are shown on the dice, which will occur with a probability of 1/21. A result of 0 can happen with 6 – 6, 5 – 5, 4 – 4 and so on, with probability 6/21. It is somewhat unlikely that children in Year 4 will be aware that the best choices of three numbers to win the bingo game will be 0, 1 and 2.

The development of the lesson describes how the children will be given three numbers, for example, 7, 13 and 20. The children are then expected to write these as a number sentence:

$$7 + 13 = 20$$

The order of the numbers as they are given provides the correct order for the addition sentence, but the children are then asked to re-arrange this to make another addition sentence and two subtraction sentences:

$$13 + 7 = 20$$
$$20 - 7 = 13$$
$$20 - 13 = 7$$

This activity is designed to develop in the children an understanding of the inverse relationship between addition and subtraction, and in this example Sharon avoided the 'hidden role' example of 10, 10 and 20. In terms of children practising the subtraction operation themselves, the examples they will use are to be drawn from a set of laminated number cards which Sharon provided. However, the lesson plan offers no examples of these and it is therefore not clear whether the numbers are selected randomly or according to a particular criteria.

Lesson four builds on the previous lesson by moving the children on to subtract 2-digit numbers. The introduction enables children to learn how to partition numbers into their tens and units, with the explanation that this will enable them to subtract more easily when they have two 2-digit numbers. The plan describes how Sharon will write a 2-digit number on the whiteboard for the children to partition, but does not indicate which number is used. This is unlikely to cause the children confusion, although it is possible that the chosen number may have zero in the units, or the chosen number may have the same digit in both the tens and units. However, since this exercise is simply to carry out the partitioning process, the impact on the subsequent subtractions is likely to be minimal at this stage.

The lesson then develops by Sharon providing a subtraction calculation which involves two 2-digit numbers and her intention is that the numbers should both be partitioned so that the tens and units can be subtracted separately and the answer found by recombining the partitioned subtraction results. Some consideration of the numbers to be used is essential here, since the digits in each column of the number to be subtracted need to be less than or equal to the respective digits in the first number. Sharon's choice of calculation is 22 – 11, one which she used in a previous lesson.

In line with examples from earlier lessons, Sharon has chosen digits for her subtraction calculation that are limited in more than one way. First of all, the solution to the problem, the difference, is the same as the number to be subtracted, the subtrahend (22 – 11 = 11), which is another example where the role of the variable is obscured.

Secondly, if the purpose of the lesson is to demonstrate subtraction using partitioning, then it would have been a more effective example if the digits in each column did not match. Following the partitioning process, the children would have to calculate 2 – 1 for the tens and also 2 – 1 for the units. This could lead children to think that partitioning only works when the tens and units digits match, with all such calculations being of the form AA – BB, and in the case of the given example in the lesson, AA – BB = BB. A further consideration is that the example 22 – 11 can be completed using a halving procedure if children know that 11 is half of 22 and so partitioning is irrelevant. This makes the example unsuitable for the objective.

For the fifth lesson, Sharon's carousel of subtraction methods continues and she introduces a method which requires use of number lines to calculate the answers to the subtraction problems by jumping, starting from the number to be subtracted and ending on the number to be subtracted from. In fact what Sharon is doing is subtraction as a difference between two numbers, solved by adding on from the smaller to the larger. In the teaching input for the lesson, Sharon begins by choosing 20 – 5 and using an empty number line to mark the two numbers in a way that represents the difference between them. There is no indication that Sharon follows through the calculation with this example, but uses it merely to enable the children to place the numbers correctly on an empty number line.

Later in the plan, Sharon chooses to demonstrate 20 – 2 by 'jumping' from 2 to 10 and then from 10 – 20 and adding together the amount of the jumps to obtain 20 – 2 = 8 + 10 = 18. This example seems to be one where the choice of numbers does not lead to the most efficient method. It would be more reasonable to use a number line for 20 – 2 if Sharon had marked 20 on the number line and asked the children to count backwards '20, 19, 18' to arrive at the correct answer, another instance of using a procedure where another would be more suitable (Rowland *et al.*, 2009).

In a later example in this lesson, Sharon chooses 20 – 8 for the children to calculate themselves using the jump method. This is a better example since it is more likely to be calculated by jumping from 8 to 10 and then from 10 to 20, giving 20 – 8 = 2 + 10 = 12. Counting back from 20 to 8 would be less efficient in this case, unless the children were able to 'jump' in reverse, jumping from 20 to 10 and then either jumping or counting back from 10 to 8. This would give 20 – 8 = 10 + 2 = 12 which will also emphasise the commutative property of addition and the fact that jumping can go in either direction to calculate the difference between the two chosen numbers.

#### 4.3.2.2a Middle attaining trainees – Dawn

Turning now to the trainees identified as being of middle ability in mathematics, Dawn provided lesson plans and worksheets for a series of lessons with her Year 4 class on fractions. In her first lesson, Dawn uses 12 marbles in a pot as a base quantity for calculating fractional amounts. She begins by discussing half of 12 and looks for the children to offer methods of obtaining the answer of 6 marbles.

The lesson then develops by removing a third of the marbles that remain, that is, a third of 6 which is 2. The remaining marbles at this point are then used to calculate a quarter of what remains, which will be a quarter of 4, which is 1. The choice of example is significant here in that the starting number needs to be chosen so that there is an integer amount for each fraction that remains at each stage.

For the children's independent work, Dawn had obtained two worksheets from the Skillswise website, entitled 'Finding Fractions', (BBC, 2010) each with a set of answers. It appears that Dawn had not made any attempt to modify the examples on the sheets, thereby implying she was confident that the examples would enable to children to meet the learning objective for the lesson. The opening question on the first sheet asks how many quarters there are if 5 cakes are cut into quarters. A diagram is provided which enables the solution to be found by counting the drawn sections rather than calculating 5 divided by ¼. Perhaps the question would have been better as 'find ¼ of 20' which gives the answer 5 and which matches more closely the objective of the lesson.

The next question is about 3 pizzas being cut into quarters, with each person getting 3 pieces of pizza. The question asks how many people could share the pizzas and as in the first question, provides a diagram showing the pizzas marked in quarters. This enables the children to count out how many sets of 3 they can find, without necessarily linking this to the calculation 3 divided by ¾ which has the answer 4.

Question 3 on the first sheet offers three prices for different size pizzas, also offering the option to buy half a pizza for half price and a quarter of a pizza for a quarter of the price. The calculations required then become finding a half and a quarter of £2.40, £1.80 and £0.60 (which might have been recorded as 60p). The question follows up by asking how many mini-pizzas have extra tomato if ¼ of the 20 mini-pizzas have extra tomato and finally, what fraction of the mini-pizzas does not have extra tomato. This question is about finding fractions of money and numbers of mini-pizzas but the choice of pizza is perhaps unfortunate as it reinforces the notion that fraction work involves pizzas in some way, whatever the calculation. Watson and Mason comment on the use of pizzas for fraction work, pointing out that:

> An image of a pizza provides a source of simple examples for adding and subtracting, maybe even for other numerical operations with fractions, but becomes unwieldy for denominators that are not closely related multiplicatively. (2005, p.95)

The second sheet starts with a question involving pictures of 4 glasses of drink and the child is asked to identify which of them is half full, ¼ full and ¼ removed. The second question asks for a 'halfway mark' to be drawn on pictures of lengths of wood, but with the instruction: 'Do <u>not</u> use a ruler', suggesting use of estimation skills rather than finding a fraction of a measured part.

Moving on to Dawn's second lesson, her lesson plan sets out to identify the location of the fractions ½, ¼ and 1/3 on a number line by attaching fraction cards in the appropriate place on a number line that goes from 0 to 1. The plan describes how ½ will be placed first, followed by 2/4 and then 3/6.

Dawn's instruction to herself on the plan is to check whether the children realize if these are the same as ½, although it is likely that once the children attempt to place the cards, it will have become apparent if they have placed the cards correctly. The activity continues with placing 1/3 on the number line, discussing how to be sure this is correct and to talk about the numerator and denominator.

Dawn is ensuring through these activities that children experiences what might be called 'obvious examples', certainly she is choosing common simple fractions to engage children with the concept and focusing on enabling the children to embed their understanding of the place of simple concepts on the number line. However, by limiting her choices to simple examples only, Dawn is ignoring the advice of Watson and Mason, who argue that:

> Novices are likely to need several teacher-provided examples so that their attention is drawn explicitly to important dimensions of possible variation, rather than distracted by unimportant ones. (2005, p.106)

The next part of the lesson changes the focus from the number line to fractions of shapes. Children are instructed to draw a rectangle of any size on a piece of squared paper and colour in a quarter of the squares. Then they compare the total number of squares with the shaded squares to form a fraction. By comparing the results from different children, different equivalences for ¼ will be discovered.

Finally there are two differentiated sheets of shapes for the children to colour various fractions. Each sheet starts with examples where the shape is divided into the same number of pieces as the denominator, for instance shading 2/4 on a square that is divided into 4 equal squares, or 5/6 of a hexagon that has been divided into 6 equal triangles.

Later examples are illustrated with shapes divided into more than the denominator, but always a multiple of it, for instance, ¾ of a rectangle divided into 16 equal squares, or 3/7 of a 14-gon divided into 14 equal segments.

In the third lesson, Dawn only gives one example on the lesson plan but a number of examples are given on four differentiated worksheets. The lesson example is related to the objective 'identify equivalent fractions', and a productive activity for the children might have been to generate their own fractions with different denominators, seeing if they can find the smallest and largest possible for a given denominator. This notion of learners generating their own examples is at the heart of Watson and Mason's recent work, and they point out from their own extensive research in this area that: The generation of examples of questions, techniques, actions, notations and mathematical objects by learners provides the material for the lesson (2006, p.24).

Throughout many of the examples that Dawns uses on her worksheet, the language is crucial, and she perhaps needed to ensure that her examples were not only sensible in terms of mathematical development but also that there is no ambiguity in the wording so it is clear to learners what they are trying to do. The second worksheet is for children of higher ability than the previous one, for example, there is a question involving colouring 7/8 of a rectangle 6cm by 4cm drawn on squared paper, requiring one of the dimensions of the drawn rectangle to be subdivided to achieve the 7/8.

Further questions on the sheet include finding the length of a pencil which started at 20cm but has reduced to 0.8 of its length after a month, and finding what fraction of a cake remains after giving 0.75 of a cake to some friends. Both of these examples are ones where the language can be described as eccentric, since in everyday use, decimals are not usually used to refer to amounts of something.

The third sheet is designed to be the most challenging and the word problems are much longer and include larger numbers. For example, this includes a question which involves 60 stickers being shared between friends so that each friend receives 0.2 of the stickers. This could be ambiguous since each friend could receive 0.2 of a sticker each, hence the calculation could be $60 \div 0.2 = 300$, meaning that 300 friends can each receive part of a sticker. However, it is likely that the problem was to find 60 x 0.2 and that each of 5 friends could receive 12 stickers. A further issue with this question is the language being used, since in everyday use the question would ask for two tenths (or one fifth) of the stickers, or even 20% of the stickers. Asking for 0.2 of a total number of items is not useful since it is not a standard way of asking such a question, as with the examples in the previous paragraph.

The final problem on the third sheet involves a recipe, starting with 100g of flour, half that much in sugar and the same amount in butter, with the remainder taken up by eggs and dried fruit. The problem asks for each ingredient as a fraction and decimal of the total as well as the total weight of certain named ingredients. The difficulty of this problem is recognized by the suggestion that a calculator may be used to help with solving this.

#### 4.3.2.2b Middle attaining trainees – Rachael

In the case of Rachael, her plans were for a Year 3 class working on addition and subtraction. The first lesson begins with adding 10 to various 2-digit numbers , then as the lesson moves from the teaching input to group activity, one group (more able) are asked to use two dice to roll two digits, which they then combine to form a 2-digit number. They then proceed to add and subtract 10, 20 and 30 in turn. This generates numbers from the following restricted set:

> 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26,
>
> 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46,
>
> 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66.

Adding 10, 20 or 30 to any of these numbers will always result in a 2-digit number. Subtracting 10 will lead to a 2-digit result as long as the number does not start with 1, similarly subtracting 20 will lead to single digits if the starting number starts with 20 and for subtracting 30, the numbers starting with 3 will give single digit results. A problem will arise if trying to subtract 20 from numbers that start with 1, or if trying to subtract 30 from any number starting with 1 or 2. It is not clear from the lesson plans whether Rachael was aware of this and gave children instructions as to how to arrange the rolled numbers.

In Rachael's case, she has resorted to the random generation of examples, as was the case with other case study trainees, although as Rowland *et al.* remark:

> Examples being randomly generated, typically by dice… may have a useful place in the generation of practice exercises, but it is pedagogically perilous in the teaching of procedures or concepts, when, as we have argued, it is simply not the case that any example is as good as any other. (2003b, p.245)

The second lesson explores 2-digit numbers and revise adding 10 and 20 to 2-digit numbers before moving on to look at adding and subtracting involving 3-digit numbers. The opening part of the lesson asks children to think about what happens when a 2-digit number is reversed; the example given is 45 becoming 54. Rachael asks what the difference is between the numbers when the digits are reversed but the lesson plan fails to indicate whether she goes on to explore this and enable the children to discover that the difference between the digits gives the multiple of 9 that result from the difference between the 2-digit number and its reverse.

The main part of the lesson starts to consider adding multiples of 10 to 3-digit numbers, with the first example given as follows: 'Introduce a low 3-digit number. Ask the children how they could find the answer to this problem, e.g. 112 + 10 = __, focusing on the tens and units.' The idea of a low 3-digit number could be perceived as ambiguous, as it is unclear whether the number 112 is low because it is amongst the first few 3-digit numbers available, or maybe it is low because each of the digits is low in numerical value. The instruction to focus on the tens and units perhaps indicates a lack of understanding on Rachael's part, since addition of a multiple of 10 will make no difference to the units and will not affect the hundreds unless the sum of the tens in each number causes a '*carrying*' into the hundreds column.

The opening part of the third lesson reviews children's learning in adding and subtracting multiples of 10 to 2- and 3-digit numbers before moving on to focus on subtracting multiples of 10 from 3-digit numbers. As with lessons from earlier cases, the problems for the calculations are derived by using dice to generate the digits. This random element will restrict the range of possible problems unless the dice has 10 faces and uses all the digits from 0 to 9.

Lesson 4 extends children's experiences of adding tens to 2- and 3-digit numbers, with the most able children using 4-digit numbers, although this is not included in the lesson objective. The activities for each ability group set out the differentiation for the lesson and include some examples of the numbers to be used. The least able group, named the 'circles', worked with the teacher assistant to add 10 to 2-digit numbers using cubes to help them. No particular examples are given, so it is assumed these were chosen by the teaching assistant. The next group, 'diamonds', used a dice to generate 2-digit numbers to which they then added multiples of 10, supported by Rachael. This is another instance of generating random examples, the dangers of which are outlined previously.

The 'squares' group was provided with two pots of number cards and was given instructions to take a card from each pot. The first pot contained 3-digit numbers and an arithmetic symbol, for example 345 + or 345 - , the second card was a multiple of ten to add or subtract to or from the 3-digit number. The range of multiples of ten and the selection of 3-digit numbers could offer calculations that cross the hundreds boundaries but this would not be guaranteed unless the sets of cards were carefully chosen.

Finally, the most able group, 'triangles', were given a set of digit cards from which to choose four at random and make a 4-digit number, for example, 1456 (or 4615, 6154 and so on). They were asked to add 10 or multiples of ten to the first five numbers they made and subtract multiples of ten from the next five numbers made. The range of possible examples is very wide here even if the digits can be chosen from 0-9 with no repeats and with 24 different arrangements of the chosen digits. It could be difficult for Rachael to limit the examples to particular instances, for example, crossing hundred boundaries.

The fifth lesson is designed to build children's mental arithmetic ability for adding or subtracting one more or less than a multiple of ten by adding or subtracting the multiple of ten first and then adjusting the additional unit appropriately. It starts by reminding children of number bonds to ten which seems an easy activity compared to what will follow. The only example given for this part of the lesson is that children will be given 8 as a starting number and are required to find the missing 2 which makes the total 10.

This resonates strongly with the example of 'Naomi's lesson' in which a trainee works with Year 1 children and practised number bonds to 10. In this lesson which was reported by Rowland *et al.* (2003b), Naomi used a pre-planned sequence (8, 5, 7, 4, 10, 8, 2, 1, 7, 3) to ask different children in the class to make bonds to 10, and Rowland *et al.* consider her choice to be a good example:

> The first and third numbers are themselves close to 10, and require little or no counting. 5 evokes a well-known double – doubling being an explicit NNS strategy. The choice of 4 seemed… to be tailored to one of the more fluent children. The degenerate case 10 + 0 merits the children's attention' (p.244).

They continue their analysis and remark that Naomi's sequence showed evidence of 'conscious design', even managing to include a 'boundary', or 'degenerate' example.

The main part of Rachael's lesson begins with adding 9 to a 2-digit number, the example given being 9 + 27. This seems an unusual way to present such an addition, since children will most likely think about reversing this to give 27 + 9, adding the smaller number to the larger and more closely representing the addition of 9 to a 2-digit number. Children are asked to find the answer and Rachael then asks for them to explain their methods. This happens ahead of Rachael explaining that adding 9 is the same as adding 10 and then subtracting 1, presumably because she wants to demonstrate that her method is in some way 'better' than the methods they have used up to that point. The lesson plan gives no apparent connection with the number fact 9 = 10 – 1, although this may have been shared verbally during the lesson.

The next part of the lesson considers addition of 11 as addition of 10 followed by addition of 1, related to the number fact 11 = 10 + 1, although again this is not evident on the plan. It would appear that demonstrating addition of 11 is easier than addition of 9, since addition of 11 involves two additions, whereas addition of 9 involves and addition followed by a subtraction, however, Rachael chose to demonstrate addition of 9 first.

In the group work part of the lesson, a set of number sentences are provided for the children to work on that involve adding or subtracting 9 or 11 to or from different numbers. The differentiation involves using addition to 2-digit numbers for the least able children to addition and subtraction to 2-, 3- and 4-digit numbers for the most able. These examples are related to the example of 'Chloe' in the work of Rowland *et al.* (2009) in which she uses the hundred square to provide a number of examples for her class of the type 70 – n. However, this limits the dimensions of possible variation by using 70 each time. In doing so, the children may think the lesson is about subtracting from 70 rather than subtracting from any number.

Rachael, in trying to teach the addition and subtraction of 'near multiples of 10' might have used a hundred square to provide a spatial representation of the calculation, as did Chloe above. However, by restricting the minuend to 70, Chloe's examples would always involve moving up rows and then backwards, with no complication such as that which might be encountered in using the hundred square for 34 – 18, for example.

**4.3.2.2c Middle attaining trainees – Andy**

The final trainee in the middle ability group is Andy, who was teaching a Year 5 class and his lessons on addition and subtraction are analysed. In the first lesson, Andy starts by asking several children to draw a single digit card from a bag and as they do so, he writes the numbers on the whiteboard as a growing addition problem.

This appears to be a simple task for Year 5 children, although it is possible that Andy was working with a lower attaining group and not the whole class. The first example he gives in his plan, not based on the children's digit selections, is 4 + 9 + 8 + 3 + 4. He asks them what the answer is to their 'created' sum and how they did it. He implies from the plan that the children should use 'tricks or techniques'; to solve the problem, perhaps looking for doubles of a digit, or pairs that add to 10. He further adds that the children will be encouraged to change the order of the digits in the sum to make the calculation easier, recognizing the commutative property of addition.

Following the starter, Andy then sets up a carousel of six activities to develop children's ability to add a string of single digit numbers. In each activity he uses either the roll of a dice or the random selection of digit cards to form the sums, again demonstrating the tendency of trainees to generate examples randomly (Rowland *et al.,* 2009).

For the third lesson the lower ability group is given five numbers as in the previous lesson but this time not multiples of 10. The numbers are: 25, 73, 89, 41, 73 and the children are asked to write these down in order and then find the total. It is unclear whether the order is as written on the board or in ascending or descending order. In this set of numbers there are two whose tens add to 100, namely 25 + 89 which gives 20 + 80 + 5 + 9 and a repeated number which can be used as part of the doubling strategy (73 + 73 = 70 + 70 + 3 + 3). The numbers here seem to have been chosen to make the calculations easier and familiar to the children based on prior learning and so can be regarded as good examples.

The lesson plan then gives directions for the children to complete a worksheet containing a mixture of 3-digit and 4-digit numbers which is beyond the scope of the objectives for the lesson and probably very challenging for the lowest ability group.

In his interview, Andy described how he likes to meet the lesson objectives through building up understanding from easy examples:

> 'I think they (the examples) are important but as long as they meet the criteria and objectives that you're trying to teach, I think that's the main key, I mean, I would order them step by step so start with the simple things, build up so they understand what they're learning… I try and make it more challenging or difficult for the higher ability, which I needed to do for this class'.

The middle ability group is provided with a set of three 3-digit numbers to add by partitioning them into hundreds, tens and units, adding each in turn and then recombining to find the total. The set of numbers given in Andy's example is 365 + 462 + 758 and if this set is compared with his example of 2-digit addition in lesson 2, it interesting to see the similarity between the two sets of numbers:

$$\underline{36} + \underline{42} + \underline{75}$$

$$\underline{36}5 + \underline{4}6\underline{2} + \underline{75}8$$

In each case, the 3-digit number contains the 2-digit number with an additional digit included. This might suggest that rather than choosing the 3-digit addition to suit objectives or strategies, it has been generated from the previous lesson plan by random alteration. This is not exactly the same as generating the example randomly as discussed previously, but using a random modification to extend a set of simpler examples to make them more challenging to more able children.

Lesson 4 opens with Andy presenting an example of a calculation which includes addition and subtraction: 228 + 263 – 137. He asks for strategies to solve this calculation but the plan gives no indication of which strategy he recommends, however, as the lesson objective is to use written column methods, it is likely that the preferred method would be to add the first two numbers using a column method and then subtract the final number from the total:

$$\begin{array}{r} 228 \\ +\underline{263} \\ 491 \end{array} \qquad \begin{array}{r} 491 \\ -\ \underline{137} \\ 354 \end{array}$$

The activities which follow are differentiated and the lower ability group is given a worksheet with what are described as 'simple 2-digit subtractions and longer 2-digit sequences'. The latter has one example listed on the plan which is 35 + 65 + 45 – 67, which can be calculated most easily by noticing that 65 and 67 are close, also 35 and 45 add easily to give 80, making the calculation equal to 80 + 65 – 67 = 80 – 2 = 78.

The middle ability group is given a mixture of problems, some of the following types: 345 + 234 + 567 – 432 and 243 + 435 – ___ = 329

Curiously the first example contains four 3-digit numbers, all of which are either ascending or descending sequences of digits. Finally for this lesson, the most able children are given a worksheet with problems including the following:

345 + 234 + 567 – 432 and 1324 + 4231 + 4326 – 3452

The ascending digits are in evidence again and in the second example only small digits are used, with 1, 2, 3 and 4 being used repeatedly. There is no evidence that Andy has generated these examples randomly, but curiously, the regularity of the sequencing in the digits for each number could be interpreted as a sign that Andy was making up the numbers.

It would have been interesting to know whether the calculations that Andy produced were tested by him in advance of using them with the children to see whether there were any inherent pedagogical difficulties with his choice of examples.

In the fifth lesson which develops further the objectives of lesson 4, word problems are introduced to give a context to the calculations. The lesson plan offers two examples of the type of question to be used:

> 'The teacher has 149 pieces of fruit and eats 33.
> How many pieces will he have left?'
> 'Each apple costs 12p. The teacher sells 56 apples.
> How much money does he make?

Both of these questions represent unlikely contexts. In the first, the teacher is unlikely to either possess so many pieces of fruit or eat so much, unless the fruit in question was grapes or some small berries. In the second example, the price for an apple is very unrealistic and it is unlikely that the teacher will be selling apples to make money.

In lesson 6, which moves on from whole number addition and subtraction, Andy explored addition and subtraction of decimals in this lesson, starting with the example $4.5 + 1.8 = \square$. He used a number line marked in tenths and approached the example by considering how many tenths need to be added from 4.5 to reach the next whole number. This strategy seems to be confused with finding the difference between two decimals, perhaps 4.5 and 5.8 and it is not clear from the plan how moving from 4.5 to 5.0 will help with the addition, although the next example on Andy's plan demonstrates that 3.5 and 4.3 can be added by dealing with the units and tenths separately as follows:

$3.5 + 4.3 = 3 + 4$ and 5 tenths + 3 tenths = 7 and 8 tenths = 7.8

Following this, Andy then demonstrates a written column method for adding decimals, using the example $3.8 + 1.6$, followed by a similar method for column subtraction using the same numbers for the example $3.8 - 1.6$ and these methods are then used in the children's independent worksheets. The subtraction example is shown as $3 - 1 = 2$ and $0.8 - 0.6 = 0.2$, giving the answer as 2.2, although in this example it was perhaps either fortunate or pre-planned to have no need to decompose a unit into ten tenths to complete the subtraction.

The lower attaining group was given examples such as $4.6 + 5.7$ and $6.3 - 4.3$ to complete. The middle attaining group was given longer strings of decimals to add or subtract, such as the examples: $4.6 + 5.3 + 2.3$ and $5.4 + 4.3 + 6.5 - 6.4$, although there is no suggestion of any context, realistic or otherwise, that would require such a calculation. It might be interpreted that Andy was choosing his examples to rehearse procedures, but until the calculations are given a realistic context, some children will regard them as abstract and something they cannot connect with.

The highest attaining children are given word problems and 'larger sequences', although no examples of these are included on the plan. It could also be argued from the evidence that Andy lacks in his pedagogical content knowledge the notion of calculating a difference between two decimals by using the whole numbers as bridging points. He might therefore have used a number line to demonstrate the procedure more clearly.

#### 4.3.2.3a Lower Attaining Trainees – Victor

Finally the lesson plans of the less able trainees are considered. Victor was teaching a mixed Year 3 and Year 4 class, and his lessons on addition and subtraction are considered. This section considers Victor's third lesson from the perspective of theoretical frameworks. The lesson plan includes a 'Point for Action' as follows:

> Hopefully with the introduction of the column method the children will be able to do more working out on paper that will hopefully help them to meet the requirements outlined in the relevant statutory frameworks.

The first concern here is Victor's use of the word 'hopefully', suggesting he may have little confidence that his activities and examples will be effective for children's learning. Secondly he feels that by moving the children towards a column method, they will not necessarily understand the method, but should be able to achieve the statutory requirements, which he obviously regards as more important.

In the introduction to the lesson, Victor works through the following examples:

34 + 50 =

245 + 40 =

These examples are similar to those he used in a previous lesson and it is not obvious which methods are required to calculate these. The earlier lessons made no use of number lines or particularly 100 squares, which would have made the first calculation straightforward. However, after these calculations, Victor begins to introduce the column method, by which it is assumed he means the standard algorithm for addition as set in the PNS. He then continues with the following examples:

43 + 30 =

367 + 70 =

965 + 80 =

The first of these is of a similar type to the first example in his introduction and could presumably be calculated in the same way, thus making the introduction of the column method unnecessary. The second and third examples, whilst extending into 3-digit numbers, have the additional challenge that the tens will add to more than 10 in each case, requiring the children to carry into the hundreds column. If the method was introduced to enable children to perform the carrying operation, then it is interesting that Victor chose as his first example two 2-digit numbers where no carrying was necessary. Given his apparent purpose for using the method, he should perhaps have chosen something like 68 + 50 to ensure the tens carried into the hundreds which in this example are empty. In trying to link the examples to the objectives for the lesson, Victor has seemed to be unaware of the discrepancy in the examples he used. When asked about the way Victor chooses the numbers for his examples, he seems to imply that the numbers are not significant, but it is the context that is very important:

> I think, ... if you can make it... put it in an exciting context, I think you can do most... what some people would consider, most simplistic, maybe boring things, you know, I'm not saying they are, but erm... I think you can, you know, can teach that effectively in a practical context, 'cause I always think, you know, rather than just doing some subtractions on the board, if you, sort of, like I said, just for example like a shop context, or something like that, then... I've done that quite a few times before, and it's quite effective really.

Victor appears to be certain that context is a more significant choice than numbers in the examples, even if that has meant his examples did not help children meet the objective.

To compare Victor's examples with theoretical definitions, it seems that his first example (43 + 30) is an example used to illustrate one procedure when another procedure would be better, a method identified by Rowland *et al.* (2009). His second and third examples are being used to teach a general procedure by demonstrating particular instances, but the procedure could be developed more progressively by using examples with fewer digits in the first instance and moving on to three or more digits at a later stage, suggesting that Victor's pedagogical knowledge lacks awareness in terms of planning sequences of examples that match the lesson criteria.

#### 4.3.2.3b Lower attaining trainees – Naomi

The final trainee to consider is Naomi, also a lower attaining trainee, who taught a Year 4 class for her final teaching placement and provided lesson plans on addition, subtraction, multiplication and division which were analysed. This section will look at some of Naomi's lessons and consider the examples from the perspective of Naomi's choices. The first lesson starts with Naomi asking the children to identify the multiple of a hundred that lies between 789 and 874, then finding the difference between the numbers by calculating the difference of each from the multiple of 100, that is:

$$874 - 789 = (874 - 800) + (800 - 789).$$

The example selected here seems appropriate to the objective, given that the minuend is 800 + 74 and the subtrahend is 800 – 11. The required multiple of a hundred is therefore 800 and the children can use this as a bridging point to help with calculating the difference between 789 and 874.

It appears from the lesson plan that Naomi requires the children to carry out the calculation in numerical form either mentally or using a written algorithm, when possibly the best choice from a pedagogical point of view could have been to use an empty number line. The children are then asked to work out a number of subtraction problems finding the difference between 3-digit numbers. Two worksheets of examples were provided; the first was designed for the lower attainers and used pairs of 3-digit numbers whose difference was always a multiple of 10 or 5.

At the top of the sheet the following advice was written: 'For each question use your knowledge of multiples of 50 to help you answer the question'. The opening example was 550 – 400 which could be completed by subtracting the hundreds to leave 150. The second example extended to requiring subtraction of all three columns: 755 – 550 = 205. Example 6 on the sheet did not match the objective since it asked for the difference between 250 and 55, which are not both 3-digit numbers but which are still multiples of 5 or 10. After six examples of this type, a change was introduced with 458 – 158. This example is the first to appear which does not contain numbers that are multiples of 5 or 10, but whose difference is still a multiple of 10; in this case it is exactly a multiple of 100, being 300. Example 9 extends beyond the objective by introducing a 4-digit number and changes the pattern of differences by not being a multiple of 5 or 10. The example is given as 1054 – 452 which has a difference of 602.

The remaining examples on the sheet are all ones that have multiples of 10 as their differences but with a mixture of 2-digit and 3-digit numbers being used. In a number of examples from this sheet, multiples of 50 did not always play a role in being a helpful step to finding the differences, despite the advice at the top of the sheet. In her interview, Naomi explained how she set out her example for the top of the worksheet and the ones that followed:

> I was usually putting an example at the top of the worksheet and then the questions underneath… it would be quite random, in a way it's just the first question that comes into my head, or it's something we've covered in the lesson that I've put in at the beginning of the lesson, but then I've decided to maybe change one digit or something, just so they recognize the concept.

It seems apparent from the examples on the worksheet as described above that in changing digits for some of the children's questions, Naomi inadvertently changed the examples so that they no longer matched the objective or her earlier guidance in terms of using multiples of 50 as helpful bridging points, since multiples of 50 did not always appear in some of the questions.

The second worksheet, which was designed for the higher attainers, includes the visual representation of a number line marked in 100s from 0 to 1000 at the top of the sheet, with the advice to use it to help solve the calculations. Each example used two 3-digit numbers either side of a multiple of 100 and the instruction was given to write down the multiple of 100 that comes between the pair of numbers before calculating the difference.

The first example was 523 – 489 which could be solved by recognizing that 500 comes between the two numbers and then finding the difference between 523 and 500, then the difference between 500 and 489 and finally combining the differences for the solution. In this case the calculation becomes:

$$(523 - 500) + (500 - 489) = 23 + 11 = 34,$$

although the children were recording this pictorially on the number line rather than setting out the numerical calculation as done here.

The pattern of examples continues in a similar way for the entire sheet, with a total of ten examples, all of 3-digit numbers either side of a multiple of 100. With these examples it is noticeable that Naomi has included use of the number line to assist with the calculation.

In the second lesson Naomi begins with two 4-digit numbers being written at each end of a counting stick, 3246 and 3346. Starting from 3246, Naomi counted forward in tens until she reached 3346. She then asked the children what 3246 + 30 is, and then 3246 + 31, as well as 3246 + 29. Using a number line from 3100 to 4100, the children then work on a series of additions as follows:

> Add 30, 31 and 29 to each of the following numbers:
> 3456, 4037, 3661, 3544, 3713, 3521, 3831, 4015.

Looking at these numbers, it is apparent that they all are in the 3000s or 4000s. Further examination shows that the calculations require different skills. The 4-digit numbers are all more than 31 away from the next multiple of 100 and so adding 30, 31 or 29 to any of them will not increase the hundreds digit in any of the numbers.

This raises the issue of why Naomi chose to use 4-digit numbers for this activity when effectively the same calculations will occur in the tens and units columns if the list of numbers given had not included the thousands or hundreds digits. Adding 29 to some of them will involve carrying from units into the tens. Subtracting 30, 31 or 29 from some of the numbers will cause no need for decomposition of hundreds into tens, however some calculations, such as 4015 – 29 will require decomposition across all columns.

Lesson 3 uses an unusual approach to help children use their understanding of number to solve a particular type of problem. The lesson takes the context of using a calculator with a broken key to find alternative ways of solving number problems, a scenario which many teachers have used over the years since the introduction of electronic calculators into school mathematics lessons. The set of problems Naomi gives the children are as follows:

$$42 + 25,\ \ 42 + 52,\ \ 64 - 15,\ \ 64 - 52.$$

The question is posed as to how to solve these on a calculator if the 5 key is broken. Each example provides a slightly different context in terms of using the 5 digit, there is an addition and subtraction with 5 in both the units and tens positions. In the first example, 42 + 25 can be reworked as 42 + 24 + 1, 42 + 23 + 2, 42 + 22 + 3, 42 + 21 + 4, 42 + 19 + 6 and so on, each calculation relying on breaking 25 into a sum of smaller numbers that do not require the digit 5 in them. In the second example, the 50 needs to be broken into 20 + 30, or 40 + 10, or 47 + 3 and so on. The subtraction examples are interesting if each calculation is considered as a difference and treated by equal addition. For example, 64 – 15 can become 69 – 20 and 64 – 52 can become 72 – 60.

These four examples seem to have been selected to offer a range of possible situations, although there is a lack of variety in the 2-digit numbers chosen, with 42 being the first number in each addition and 64 as the first number in each subtraction. Further, the second numbers for the addition problems are 25 and 52 which clearly have a similarity based on the reversal of the digits. The theme of placing the 5 in all positions continues with the set examples for the children to calculate independently:

$82 + 35$, $75 + 24$, $87 + 56$, $54 + 32$, $87 - 25$, $75 - 21$, $96 - 52$, $56 - 23$.

In this set of calculations, the digit 5 appears in the tens and units position for both numbers in both additions and subtractions, whilst the remaining digits show less reliance on the same numbers. However, 8 appears in the tens place for three of the numbers used as the first in each calculation and both 75 and 87 appear twice. The more able group of children are extended by giving them two multiplication calculations and two divisions, each of which contains a digit 5 which must be replaced: $32 \times 5$, $15 \times 2$, $80 \div 5$, $105 \div 5$

From a procedural perspective, replacing the 5 in the multiplication problems makes use of the distributive law, so for example, $32 \times 5$ can be written as $(32 \times 4) + (32 \times 1)$ and $15 \times 2$ can be written $(14 \times 2) + (1 \times 2)$.

The division calculations are more challenging, since for example, $80 \div 5$ is not the same as $(80 \div 4) + (80 \div 1)$. A class of Year 4 children is unlikely to be able to make this distinction and realize that perhaps the best solution involves using equivalent fractions and writing $80 \div 5 = 160 \div 10 = 16$. The final example, $105 \div 5$ can also be treated this way to become $210 \div 10 = 21$.

In lesson 4 Naomi chooses a selection of 3-digit numbers for the children to double by breaking them into hundreds and tens and doubling each part separately before recombining to give the total. The numbers she chose had one feature in common which the lesson plan does not suggest was deliberate – each of the numbers is not only a 3-digit number but is also a multiple of 10. This means there is no need to partition the numbers into hundreds, tens and units before doubling and recombining as a total.

In the starter, Naomi uses 320 and 420 as her numbers to be doubled. In each case, there is no carrying from tens into hundreds or from hundreds into thousands and so it is likely that the doubling could be carried out by many of the class without partitioning. She then moves on to ask the children what half of 420 is, which could be completed by partitioning or by noticing that each digit is even and can therefore be halved mentally.

Finally in the starter, Naomi used 270 as a number to be doubled, which in this case requires carrying from the tens into the hundreds, worked as:

$$(200 + 200) + (70 + 70) = 400 + 140 = 540$$

The calculation ends with halving 540 by halving each of 500 and 40, giving 250 and 20 which combine to make 270. It is interesting that this example of doubling 270 and then halving 540 offers two different routes from one number to its double and back again. It is unlikely that Naomi would encourage the children to halve 540 by halving 400 and 140, even though they are the numbers reached when doubling 200 and 70.

The main part of the lesson involves the children doubling the following set of numbers: 330, 260, 350, 440, 470, 290, 130, 240, 410, 640, 720.

The more able group doubled the whole set, whilst lower ability children did not need to double the last two examples in the list, although no explanation is given as to why this is the case. From inspection of the numbers, it seems to be that the last two examples both start above 500 and so doubling will take them into thousands, involving carrying from hundreds into thousands and so part of Naomi's strategy for differentiation could be to restrict the lower attainers from attempting these examples.

However, it could be interpreted that Naomi was attempting to demonstrate that she had included differentiation in her lesson planning, even though the activity may have been one which all children could have attempted, and the lower attainers may have been able to extend their learning significantly by engaging with those final two examples. The order of the examples provides variety in whether carrying is required or not, the list for lower attaining children ending with three examples that require no carrying.

The next part of the lesson requires children to halve the following set of numbers: 340, 460, 860, 340, 780, 540, 250, 370.

The first six examples are for all of the children to work on and every number is an even multiple of ten which aids the partition process and implies that every halved number will also be a multiple of 10. The final two examples are odd multiples of 10 and so their halves will end in 5, being multiples of 5 but not 10.

In the plenary to the lesson, Naomi uses the number 730 and asks the children to halve it. Since 730 is an odd multiple of 10, it is an example that only the more able children will have experienced in the earlier part of the lesson. Naomi assumes the children will solve this by halving 700 and 30 to get 350 and 15, recombining these to get 365, but this example, in being unusual to many of the children will either be a hindrance to them as not being familiar to those they have attempted, or it could serve as a challenge to extend their learning. Whichever of these cases resulted in the lesson, the choice by Naomi was likely to have been random and therefore not chosen for pedagogical reasons.

In the next lesson, Naomi sets out a number of examples so that children can use a grid method to multiply a 2-digit number by a single digit. The starter example is 34 x 2 which is presented in grid form so it can be calculated in a similar way to the doubling and halving of the previous lesson, that is, by partitioning into 30 x 2 and 4 x 2 then recombining. The main part of the lesson offers the children differentiated sets of multiplication problems, starting with this set for the lower attainers: 42 x 2, 42 x 3, 56 x 2, 56 x 3, 28 x 2, 28 x 3, 62 x 2, 62 x 3.
These examples offer some calculations with no carrying, some with carrying from units into tens, some with carrying from tens into hundreds, some with carrying from units to tens and tens into hundreds. However, the sequence of the examples does not present the various alternatives in increasing difficulty. The common features to this set of examples are that they each have one of four 2-digit numbers (42, 56, 28, 62) multiplied firstly by 2 and then by 3, giving lower attainers practice with their 2 and 3 times tables.

The middle attaining group was given the following set of calculations:

42 x 3, 42 x 4, 42 x 5, 56 x 3, 56 x 4, 56 x 5,

28 x 3, 28 x 4, 28 x 5, 62 x 3, 62 x 4, 62 x 5.

The numbers used are the same as before, but this time each is to be multiplied by 3, 4 and 5. In this set of calculations, every example requires carrying. The highest attainers were given the following set of calculations:

42 x 6, 42 x 7, 42 x 8, 56 x 6, 56 x 7, 56 x 8,

28 x 6, 28 x 7, 28 x 8, 62 x 6, 62 x 7, 62 x 8.

The same set of numbers are used as before, with the multipliers being 6, 7 and 8 this time, which again means that every calculation includes some carrying from one column to the next, both for units to tens and tens to hundreds. The selection of numbers in these calculations again demonstrates a repetition of particular digits or numbers. This seems to be a feature of the examples provided by the Hamilton Trust, although it is not the scope of this work to analyse that scheme in depth. However, Naomi draws from it regularly for her examples and missed the opportunity to vary her examples.

Further work for the children came in the form of two differentiated worksheets, each containing a number of word problems with a multiplication calculation embedded within the worded context. The first example on the lower attainers' sheet asks for the number of pencils in a cupboard if there are 9 packs of 5 pencils. This is a realistic context and given that the numbers to be used are printed as digits in the question, it is likely that many children will be able to deduce that the solution is to be found by multiplying 9 x 5 to get 45 pencils.

It is questionable, however, whether the children will really be demonstrating an understanding of multiplication from this example, simply an ability to interpret word questions in such a way as to anticipate the likely calculation. This is another area where other research has been carried out, but this does not feature within the scope of this study.

As the examples continue on the first sheet, some of the contexts appear to be either contrived or unrealistic. For example, question 3 describes a context where there are 3 plants in the school garden, each with 7 leaves. This suggests perhaps a very poorly stocked garden and an unusual collection of plants, given that the number of leaves on the plants is exactly the same in each case which is rather unlikely.

Question 6 also raises issues about context reality and also solvability, presenting a situation as follows: 'There are 12 boys and 5 girls in each class in a school. How many children are in the school?' There are two issues here; firstly it is most unlikely that each class would have exactly the same gender split of 12 boys and 5 girls. Secondly, the example does not state how many classes are in the school and so it is impossible to calculate the total number of children.

The last two examples on the first worksheet change to a different style of question which is used throughout the second sheet, aimed at higher attainers, and so will be examined from that perspective. The higher attainers were given a sheet of examples that were of a more open nature.

For example, the first example is as follows: 'In a classroom there are __ rubbers on each table. There are __ tables and 28 rubbers altogether. What different combinations of rubbers and tables could there be?' The context is seeking a solution in which the missing numbers form the product 28 and so this could be 1 and 28, 2 and 14 or 4 and 7 (and with the corresponding commutative versions). However, the context is likely to be realistic for only some of those combinations. The 1 and 28 pairing cannot be correct since the question gives both tables and rubbers as plural quantities. The possible solutions are then:

2 rubbers on each of 14 tables – unlikely for a typical primary classroom
4 rubbers on each of 7 tables – possible if desks are grouped as tables
7 rubbers on each of 4 tables – again possible in a primary classroom
14 rubbers on each of 2 tables – not a likely situation in a classroom

Question 3 is based on the plants and leaves example from the previous worksheet, but expressed in the open-ended format as: 'A school garden has __ plants and each plant has __ leaves. There are 30 leaves in total. What different combinations of plants and leaves could there be?' This question again implies an equal number of leaves on each plant which is perhaps unrealistic and given the plural nature of each, the possibilities are as follows:

2 plants with 15 leaves each
3 plants with 10 leaves each
5 plants with 6 leaves each
6 plants with 5 leaves each
10 plants with 3 leaves each
15 plants with 2 leaves each

The question provides an opportunity to explore factors of a given number, although this is not the purpose of the worksheet from Naomi's perspective.

Her choice of example could lead to some discovery learning by the children, outside the scope of the lesson objective, but if Naomi allowed this, she may have found that some children benefited enormously from the possibilities afforded by her choice of example.

Question 5 offers another example where factors could be explored but the context within the worded question is very unrealistic. The question is set out as follows: 'At a fish and chip shop there are __ portions of chips sold and __ chips in each portion. 120 chips were sold in a day. What different combinations of portions and chips could there be?'The factors of 120 provide the range of combinations, but many of these are likely to be unrealistic in terms of numbers of portions of chips sold and the number of chips in a portion.

The final lesson starts with considering how to find out how many 6s are in 78. The approach taken by Naomi is to split 78 into 60 and 18, since 'we know that there are ten 6s in 60 and three 6s in 18'. This assumes that all the children know those facts and can identify that 78 can be split that way to produce two multiples of 6. A further example is then given which is to divide 96 by 6 in a similar way, implying a split that recognizes 96 as 60 + 36 and that there are ten 6s in 60 and six 6s in 36, making sixteen 6s in 96.

For the main part of the lesson, different ability groups are given sets of division calculations to complete. The lower attainers are given the following set:

$$75 \div 5,\ 90 \div 5,\ 105 \div 5,\ 48 \div 4,\ 56 \div 4,\ 64 \div 4,\ 42 \div 3,\ 57 \div 3,\ 69 \div 3$$

This set of examples gives the group three calculations where they are dividing by 5, 4 and 3, with a different set of numbers for each divisor. Naomi has ensured through her choice that the calculations lead to exact answers with no remainder by making the numbers to be divided a multiple of the divisor.

The middle attainers group had the following set of division calculations:

$85 \div 5$, $125 \div 5$, $72 \div 4$, $78 \div 6$, $69 \div 3$, $96 \div 3$, $96 \div 4$, $96 \div 6$, $99 \div 9$, $117 \div 9$

In this set, the divisors vary between 3, 4, 5, 6 and 9 and again the numbers to be divided are multiples of the divisor and so there are no remainders. One example appears in both groups' lists, namely $69 \div 3$. The use of 96, as a number with many factors, provides three different examples, being divided by 3, 4 and finally 6, with the final example moving beyond the lesson objective of being able to 'divide two-digit numbers using chunking' by providing an example of a 3-digit number to divide.

The highest group was given a further set of division calculations, as follows:

$135 \div 5$, $160 \div 5$, $96 \div 4$, $96 \div 6$, $96 \div 3$, $126 \div 3$, $108 \div 6$, $132 \div 6$, $126 \div 9$

In this set of examples, most involve 3-digit numbers, taking these examples outside and beyond the specified learning objective.

The same set of divisors is used as for the middle attainers, namely 3, 4, 5, 6 and 9. In order to use the chunking method as illustrated by Naomi, the children would need to be able to recognize suitable partitions of the 3-digit numbers into smaller multiples of the divisor. For example, 126 might be partitioned to give $90 + 9 + 27$, each part of which divides by 9 to give $10 + 1 + 3 = 14$.

### 4.3.3 Connection

The next dimension of the Knowledge Quartet is Connection, and analyzing the case study data in respect of this dimension provides evidence of a range of approaches across the case study sample. The evidence for trainees using aspects of the connection dimension is taken from the interviews with the case study trainees. Taking the more able trainees first, it is noticeable that both Suzy and Sharon were aware of the notion of making decisions about sequencing when planning to use examples in their lessons. For example Suzy, when talking about choosing examples for teaching the whole class, said: 'Usually I have one that I've written on my plan to start with, and I'd see how they go, and then see if we need to… often then I would just make them harder'.

Sharon is more concerned with how her own ability and knowledge might affect the examples, saying:

> Within maths you'll have areas that you have difficulty with and when I looked at time, I thought "Oh, where do I actually start with time? What do I do first, to lead on?" and you have to really think it through yourself before, so that might help, sort of like, sequencing.

Both trainees showed in their interviews that they are aware of complexity in their teaching examples, interpreting it in most cases as the range of differentiated examples needed to cover the ability range of the class. Sharon again provides the following explanation for her approach:

> The questions… if they are pitched too high, I just thought, they are not going to understand it, 'cause it was aimed at Year 4, but as low ability, they are not necessarily working at Year 4 level, so it was just too hard.

The trainees regarded as middle ability also demonstrated evidence from the connection dimension of the Knowledge Quartet, for instance Dawn, who showed an awareness of how the different elements of mathematical learning linked together. She planned the parts of her lessons by linking them together according to the concepts and processes she was teaching, making the links visible to the children: 'I would… do a wall map depending on what the focus was'. She also talked of starting simply and progressing to more complex issues in her lessons, and in one instance where the examples in a textbook were not appropriate for the children's ability, she said:

> Sometimes I have probably substituted simpler ones to start with and then used… you know, the ones that were given. Maybe just sort of things that fir in better with what point I'm trying to get across.

Later, in referring to worksheets that she made for her class, Dawn described that she would: '…again start with some simpler ones and move onto more complex ones'.

This approach was echoed by Andy who, in deciding which examples to use in his main teaching input, recalled that: 'I'd start with a very simple one, very easy, straightforward', and then for his worksheets Andy describes how he decides on examples:

> I'd first of all judge it yourself first, will they be able to do this based on the last lesson, by marking their work and what they can do already, and then obviously as they do it as well I watch carefully then it's quite simple, it's easy to move them on, so I'm trying to do it do they can do it without needing too much help but with some challenge.

Rachael anticipates the complexity of the examples and allows for this as well as varying the level of the questions:

> I'd start with easier questions and build up to difficult ones and then maybe put an easier one again, so they weren't aware of how the order would go, like easier to harder.

Finally for this section, the lessons and interviews of the lower attaining trainees, Naomi and Victor are examined for evidence of connection from the Knowledge Quartet (Rowland *et al.,* 2009). Starting with Naomi, she described in her interview how she links the mathematics topics to a range of possible teaching ideas:

> Well, I'd read it (the Hamilton Trust plan) first, see whatever subject, what part of maths I'm teaching and I'd read that, then I'd probably think right, well I can do this, and probably do like a cloudburst with different ideas, obviously I'd have to know what the ability of the children are, and that's what would probably influence me.

Later, when discussing items from one of her lesson plans, Naomi demonstrated an awareness of the need to link examples in order to make a steady pedagogical progression for the children, but in the context of making the connections between examples obvious for children, rather than connect the mathematics with other topics:

> If you look at the most recent one that they've done, (950 + 50) so because this is a harder question to answer, trying to link the two, I thought that if I did them together, then they are still going to understand how when you add something to 650 to get 1000, it's going to be the same as taking away 650 from 1000. So it was making a connection between the two, but with the class, they found it really hard to think of something and then go on to the next idea, there needed to be some sort of link between them.

Victor seemed to be unsure about the idea of connection, also referring to connecting the examples within a topic. He suggested that his choice of examples was influenced by the approaches the children would use in order to engage them in the learning:

> I like to get the children involved quite a lot, so I try to get the children involved and do the job at the same time... erm... the other one is they've got to be practical and effective and fun really, you know, they are the three things I sort of always go for.

Later, Victor described how he likes to connect the examples in a sequence, particularly for worksheets:

> There always tends to be a sequence, it starts off fairly simple and easy, and then it gradually gets a bit more difficult, the towards the bottom there's always some other activity, or once you've finished that there's always some other activity that draws it all together, probably a sort of classic, linear approach really.

The evidence from these trainees seems to suggest that they have limited notions of connectionist teaching as described by Askew *et al.* (1997) and relate connection only to the sequence of examples, and almost always from simple to more difficult, whatever that might mean for them, although this was not explored in the interviews.

### 4.3.4 Contingency

In terms of the final dimension of the Knowledge Quartet, namely contingency, the evidence is taken from interviews with the case study trainees. Starting with the higher attaining cases, both Suzy and Sharon made use of opportunities in their lessons to extend children's learning by making use of a range of methods.

Sharon described one situation from her placement where she used a display of nine questions in a grid which she used as an interactive display for the children to select their own set of examples. Suzy recalled a lesson where she observed the class teacher and then emulated him in teaching a lesson on measurement, bringing in lots of ideas and activities and enabling the children to learn through discovery methods. This lesson, as described by Suzy, would have enabled the children to make connections between different contexts, in the style of the connectionist teaching identified by Askew *et al.* (1997).

The idea of contingency was rather less evident in the data from Victor and Naomi, with Victor recalling how his lesson plans often needed to change due to not covering everything as quickly as he had hoped. This is not strictly in line with the ideas of Rowland *et al.* (2009) who saw contingency as the way trainees responded in the classroom to learning opportunities that arose unexpectedly from the children. Victor expressed a contingent approach as the way he adapted his plans through the week, although he seemed muddled in his description in terms of timescales:

> You know, 60% I did my own (examples) because I'd look at it sometimes and say 'That's not right' or... because it's quite an intense schedule, like it's five days, one lesson every week, obviously you're not going to get everything done in one lesson, so you sometimes spend half the week doing one lesson.

He had, earlier in the interview, suggested that:

> If it's a one-off (lesson), then you usually know what the children are doing at the moment, at the current time, er... sort of in my head, I have built up a good stock of erm... warm-ups you can do, related to the task, and I know it doesn't have to be totally related, but it's always good.

In this, Victor seems to be suggesting that he is able to select activities on the spur of the moment, but perhaps the contingency situation is one where he is asked to take a lesson at short notice, rather than respond to children's learning at key moments.

Naomi expressed some evidence of contingency in terms of dealing with the situation when children work more quickly than expected and complete the set tasks. She appeared to adopt a strategy of asking the children to generate their own examples, which is an aspect of learning mathematics which Watson and Mason (2005) expound as being effective on many levels. Naomi specifically mentions the higher attaining children in this context:

> We were doing problem solving and, especially with the higher attainers, if they finished their work, which they usually did very quickly, … I was getting them to think up questions of their own, I mean their own problem question… that they could pass to their partner and then they had to then explain to the class, give their example to the class, of the question they'd made up.

It could be interpreted that Naomi's activities for higher attainers are not challenging, but she engages them in generating examples which is a useful learning tool.

### 4.4 Types of Examples

In the review of literature, Watson and Mason (2005) presented a set of types of examples, which are reproduced again here:

- Illustrations of concepts and principles
- Placeholders instead of general definitions and theorems
- Questions worked through in textbooks or by teachers
- Questions to be worked on by students (exercises)
- Representatives of classes
- Specific contextual situations

Each of these types of examples have particular features and represent different uses and purposes of examples in teaching and learning mathematics, and in this section evidence is analysed to see the extent to which the trainee primary teachers use different examples within these categories for their teaching.

### 4.4.1 Illustrations of concepts and principles

Watson and Mason (2005) suggest that these might include demonstrating equivalence of fractions by using two particular fractions that have that property. From the data collected in this study, a number of trainees were identified as using these examples in their teaching.

In Sharon's placement, she was teaching subtraction to Year 4 pupils and in one instance she describes how she chose some examples purposely to set up a situation where the pupils were able to choose some of the numbers in the examples they were completing:

> I was doing subtraction… and they had to subtract a number below from the higher number, so I deliberately chose, like, 99 and all the digits were lower, 'cause the teacher was in the lesson and said to me… 'Are you not worried that they are going to start having a bigger number?' and now I've covered that, so I'd already covered the fact that they would then be able to just pick out a number at random, 'cause it's a bit more fun, and I knew they wouldn't have a number that was higher than a 9, so I did.

The lesson builds on that of the previous day which considered the extended column method for single and 2-digit numbers, such as 27 – 3, but in this lesson both the numbers in the calculation have two digits.

It should be pointed that 27 – 3 would not be a suitable calculation for the extended column method, and should be carried out either mentally or by counting back on a number line or 100 square, making this an example where the chosen procedure is not a sensible one, as described in Rowland *et al.* (2009).

For Sharon's input, she describes how she will use 2-digit numbers to show the children how to set out the working but without listing a specific example. However, for the children's activity to consolidate and practise the learning, Sharon introduces the number 99 which will be used as the first number in each calculation the children work on, giving each example a specific form to enable children to work on the particular procedure of subtraction. By using 99, Sharon is ensuring that the children will always be able to subtract any 2-digit number without any need to 'borrow', 'decompose' or as Sharon describes it: 'knock on next door'. The children are then able to make calculations by drawing two cards from a set of single digit cards to make a 2-digit number and subtracting that from 99 to find an answer. It is unclear whether 0 or 9 feature in the sets of cards, since the inclusion of these digits could lead to some children calculating 99 – 00, 99 – 01, 99 – 02 and so on, or 99 – 99, which are instances of boundary examples (Watson and Mason, 2005).

It can be concluded that by using 99 as the starting number, Sharon is using what Watson and Mason (2005) might describe as a mathematical object from which learners might encounter specific contextual situations. Rowland *et al.* (2009) might put this into their category of examples for teaching a particular instance of a concepts or procedures.

Sharon recalls this particular choice of example in her interview, explaining that the choice is 'really, really important' and justifying the choice by saying:

> They had to subtract a number below from the higher number, so I deliberately chose, like, 99 and all the digits were lower,'cause the teacher was in the lesson and said to me, he was like, 'Are you not worried that they are going to start having a bigger number?' and now I've covered that, so I'd already covered the fact that they would then be able to just pick out a number at random, 'cause it's a bit more fun, and I knew that they wouldn't have a number that was higher than a 9, so I did...so I think with that, if I'd have not thought of that, they would have come across a lot of problems, because they hadn't actually come on to the fact of subtracting of 10 and I wasn't going to cover that with them because they were struggling anyway so we were going to leave that, but you do have to think about it, and if you don't challenge them enough then they can't do it.

One of Rachael's lessons begins with adding 10 to a single digit number, for example, 4 + 10, and children displaying the answer on number fans. This choice of resource enables the children to recognise the symbolic representation of adding 10, putting a digit 1 in front of the 4 to make 14, making use of a theoretical approach described in Liebeck (1990) and moving the children from experience and language to a pictorial and then symbolic approach to recording their learning. This does not allow Rachael to determine whether the children have understood the operation of addition of 10 as meaning 'to put 10 more with the set of 4'. This suggests the procedure being used was not the most sensible for the concept being developed, an instance of the type described by Rowland *et al.* (2009), and a reasonable interpretation of this situation might be that Rachael needed to allow the children to have more experience of this type of calculation, using real objects and recording in their own pictorial system.

It might also be argued that in the starter part of a lesson, the children should be rehearsing something they already understand, but in this lesson the activity comes after the starter and at the beginning of the main teaching input.

The lesson continues with adding 10 to various 2-digit numbers which may seem to be a better activity as it involves changing the tens digit for one which is one more than what was there previously, for example in 53 + 10, the 5 changes to a 6, representing the change from '50 and 3' to '60 and 3'. For children who have mastered this concept, Rachael moves on to ask them to add 20 or 30 to her given 2-digit number, although there is no evidence that she chooses numbers above 80 for adding 20 or above 70 for adding 30, either of which would require the children to add across 100. The 100 in this instance acts as a kind of boundary, since the conditions either side of it have significant differences in terms of the calculations.

Andy talks about finding pairs of multiples of 10 that total 100, for example, 10 and 90, 20 and 80 and so on, and that the children should use these to help them total their sets of numbers. He also reinforces the use of doubles such as 20 + 20 = 40 as a further addition strategy, encouraging the children to make use of sensible procedures where possible and PNS approaches when it is helpful to do so. For more able children, Andy sets out numbers that are not multiples of 10, his first example being: 36 + 42 + 75.

The discussion around strategies leads to resetting the problem as follows:

$36 + 42 + 75 = (30 + 40 + 70) + (6 + 2 + 5) = (70 + 70) + 13 = 140 + 13 = 153.$

In this example the numbers were chosen to enable use of doubles in the calculation, but there are no instances here of pairs of multiples that add to 100.

### 4.4.2 Placeholders instead of general definitions and theorems

These are illustrated in their work by the instances of using a dynamic image of angle to demonstrate the property of angles in the same segment of a circle as being equal. In the data from this study there were no examples of this type of example being used. One reason for this could be that in primary mathematics, as opposed to secondary mathematics, there are fewer opportunities for teachers to present precise mathematical definitions through examples, and it is unlikely that many primary teachers, particularly trainees, will introduce their pupils to theorems of any sort.

### 4.4.3 Questions worked through in textbooks or by teachers

These are sometimes also referred to as 'worked examples'. These are often presented in secondary or higher mathematics as a way of demonstrating model answers to questions, perhaps in preparation for examinations, where the layout and stages of a solution might be awarded partial marks. In primary mathematics this is likely to be a far less prevalent practice, although the examples used by teachers in the whole class input of their lessons may give rise to opportunities to set out a complete solution to an example in order for pupils to follow the line of reasoning to develop their understanding of a concept, although this seems to overlap with the first category of example in this typology as described earlier.

There were very limited instances from the data which could be argued to fit entirely within this category. Naomi's lesson plan did offer one such instance which is worth analysis. Her lesson had the objectives: 'To be able to understand that when multiples of 10 go over 100 then they are 'pushed over'' and 'to be able to estimate to the nearest 100 before adding two 3-digit numbers'.

In the oral and mental starter for this lesson, Naomi chose 482 + 335 as her opening example, asking the children to calculate this in pairs and then checking whether the answer (817) is reasonable. There is no indication as to how the children should decide what constitutes a reasonable answer, although presumably it can be interpreted that if they calculated the correct answer, it should be considered reasonable. The follow-up question from Naomi asked why the total is over 800, and the lesson plan suggested the explanation is in terms of the tens totaling more than 100 and so the total is 'pushed over' into 100. The phrase 'pushed over' is not recognised mathematical vocabulary from the National Curriculum (DfES, 1999) or the PNS (DfES, 2006), but Naomi had presumably explained the phrase during her lessons. Had additional data collection been possible, through classroom and lesson observation, then it is possible that specific instances of this type of example could have been recorded and analysed.

### 4.4.4 Questions to be worked on by students

These are designed to give pupils the means to use, apply and gain fluency with techniques. These were far more common amongst the data collected from the primary trainees in this study. Instances of these are now analysed, both as direct examples and comments about trainees' approaches to using them.

Whilst teaching on her final placement, Suzy was teaching a grid multiplication method and her lesson plans provide some evidence of how she chose a simple example as an illustration to begin with and then extended the difficulty for more able groups or individuals, providing a worksheet from which the children can practice. In one particular lesson Suzy develops the notion of multiplication using a method which leads children to use the standard algorithm for long multiplication.

The lesson plan outlines the stages by which this will happen and then makes reference to a worksheet of various examples to allow children to practise the method, in the form of an 'exercise' as described by Rowland (2008). This worksheet can be analysed in terms of its examples and the sequence Suzy has selected to present them.

In the first part, two examples are set out to demonstrate the grid and column method for calculating 234 x 6. The grid element provides a structure by which to carry out the multiplication of each digit in 234 by 6 and the column part of the method gives a structure for adding the resulting products to form the total. It is set out as thus:

**234 x 6**

| x | 200 | 30 | 4 |
|---|---|---|---|
| 6 | 1200 | 180 | 24 |

| Th | H | T | U |
|---|---|---|---|
| 1 | 2 | 0 | 0 |
| | 1 | 8 | 0 |
| | | 2 | 4 |
| 1 | 4 | 0 | 4 |

234 x 6 = <u>1404</u>

The choice of example here does not seem to cause any unusual problems with the calculation or the details of the method. By choosing 2 in the hundreds for 234 means that when multiplied by 6, the result will be a 4-digit number. This would be the case for any choice of hundreds except 1, which would lead to the partial result of 600 from that line of the calculation. With the tens and units of 234, any choice above 1 will provide a 2-digit result to be added into the total. On this basis, a good interpretation of this would be that the example used to demonstrate the method on the worksheet is a good choice from a pedagogic perspective and the possible range of variation (Bills *et al.,* 2006) in the choice of digits is narrowed to provide an example that steadily builds the number of digits in each part of the calculation.

The worksheet then has a set of seven examples for the children to work through. The first of these is 453 x 2, which is different to the demonstrated example in that the choice of 2 as multiplier combined with the relatively small digits in 453 leads to two of the partial results being the same number of digits as the partition being multiplied. If this example is set out as required, it would look as follows:

**453 x 2**

| x | 400 | 50 | 3 |
|---|---|---|---|
| 2 | 800 | 100 | 6 |

| Th | H | T | U |
|---|---|---|---|
| | 8 | 0 | 0 |
| | 1 | 0 | 0 |
| | | | 6 |
| | 9 | 0 | 6 |

453 x 2 = 906

An issue relating to this choice of example is that doubling is a regular part of children's learning within the PNS and this example would be quite easily calculated mentally as (400 x 2) + (50 x 2) + (3 x 2) giving 800 + 100 + 6 = 906.

The use of the grid and column method is cumbersome for such an example, and perhaps is a situation where the example uses one procedure where another might have been more appropriate, as described by Rowland *et al.* (2009).

Example 3 on the worksheet continues with small digits, using 612 x 3. This will have the effect of producing a 4-digit number, a 2-digit number and a single digit from the partial results. Children may have difficulty with this as there is a break in the apparent sequence with there being no 3-digit number to include in the final column addition.

The remaining examples all fall into a similar category, namely that the partial results formed are not necessarily similar in terms of the number of digits they contain, caused by the choice of digits in the multiplicand and the multiplier. It would seem that this worksheet provided an example 'that everybody could actually work through' followed by a selection of others which could be tackled by pupils of different abilities. Suzy appeared to be confident in her choice of examples on the grounds of pupil success at completing them correctly, a deduction made both from the evidence of her lesson plan and her subsequent interview.

By examining the transcript of Sharon's interview, it is also possible to gain insight into her approach when choosing examples for children to work on independently.

When asked about the factors involved when choosing examples, Sharon responded with:

> I normally start off quite easy because it's better to start off with a really easy one that they can do, because they already get the confidence up that they can do it…if they can't do what you think is easy, 'cause you can sometimes pitch it too high, if you think that's easy they can still struggle with it, so then you can lower it or higher it as you need to.

This approach was also evident from Sharon's description of how she might choose examples specifically for a worksheet or a set of problems for the IWB:

> It often took quite a while to explain it, we would do a lot on whiteboards and then they'd have a sheet and that would be about it, but in my own planning I would always write out, so instead of using a worksheet I wrote the questions on the board sometimes, so then I could go to the children who'd finished and write questions in their book, and I would get more…progressively more difficult.

To sum up at this point, both Suzy and Sharon, who are more mathematically able trainees, seem from their lesson plans and interviews to be confident enough to plan lessons without resorting to following PNS guidance too rigidly, and choose examples by ability of pupils, but always ensuring that the first examples can be tackled by every pupil before setting examples which cater for the range of abilities, which matches the findings of Bills and Bills (2005).

Using evidence from one lesson plan, Dawn gives a number of examples on two differentiated worksheets. The examples are related to the objective 'identify equivalent fractions', but starts with a place value discussion to identify the value of each digit in 375. This establishes that the 3 represents 300, 7 represents 70 and 5 is 5 units.

The numbers 37.5 and 3.75 are then considered to introduce respectively tenths and hundredths. The development here is that children are introduced to the equivalence of 0.75 as 75/100 and that this is equal to ¾ since both 75 and 100 can be divided by 25.

The first worksheet involves matching two sets of fractions, one set being given in the form of a numerator and denominator, the second being in decimal form. The first set starts with ½, ¼ and ¾, followed by each of the tenths from 1/10 to 9/10, ending with four examples of hundredths, namely 45/100, 37/100, 15/100 and 99/100. It might be argued that the progression from simple to more complex is evident in these examples.

The second worksheet uses fractions of amounts in the context of word problems, linking fractions with division. The opening question is worded thus: 'Paul has 24 marbles. He gives ½ of the marbles away. How many marbles are left?' This example links finding a half with dividing by 2, although the solution could be given as '1/2 of them' which would be correct but not involve the division calculation. The second question is more complex:

> Kadin draws a rectangle which is 3 squares by 4 squares'. He colours in ¼ of the squares. How many squares are coloured? What fraction of the squares are not coloured?

The first part of the question requires use of multiplication to determine how many squares there are in the rectangle, in other words $3 \times 4 = 12$. After this the division of 12 by 4 can be carried out to determine how many squares are coloured, $12 \div 4 = 3$.

This leaves the option of linking multiplication and division as inverse operations although it is not clear whether this link was made by Dawn in her teaching. The final part of the question is grammatically incorrect since a single fraction is required, hence the wording of the question should end with *'...is not coloured?'* The calculation here, involving knowledge of fractions that total 1, is 1 – ¼ = ¾.

Question 3 tells us: 'Daisy shares 30 marbles between her friends. Each friend receives 6 marbles. What fraction of all the marbles does each friend receive?' A less complex question again, requiring no calculation, only the awareness and knowledge that sharing equally between 6 will give each 6/30 = 1/5 of the total. There is the possible issue of whether Daisy keeps any marbles herself although the wording of the question makes it difficult to determine this.

Question 4 offers the following:

> Jack has a piece of string which is 50cm long. He cuts it into two equal pieces. How long is each piece? What fraction of the string is each piece? Now write this as a decimal.

This question requires division of 50 by 2 in the first instance, followed by recognition that dividing by 2 is the same as finding ½ and finally knowledge that ½ = 0.5 and how to write this. The development of these questions shows that Dawn's approach is reflected in her lesson plans and the examples she uses for the pupils, although she makes little use of equivalent fractions. Firstly she matches the lesson objectives with the PNS and secondly matches the ability levels of the pupils, starting from easier examples and moving towards more difficult ones, supporting the findings of Bills and Bills (2005).

A similar view on choosing examples was expressed by Rachael who, when considering the range of abilities for the children in her Year 3 class said:

> Because of the abilities, I found it really important and I observed that the teacher had said that some of the examples were for a particular group, so they'd be a bit more aware that that question was for them, and a lot of the time, they'd do a bit of the examples and the lower ability group would be sent off to do their own with the teaching assistant or another teacher.

When further questioned during her interview about how the examples for her lessons were chosen, Rachael expanded on her previous comments:

> I'd literally just go to the Mathszone and go through them, and some of them were either not suitable or too difficult and then I'd choose ones that I thought were good. If I was just doing examples that I was just drawing on the board then I might think about them before and jot them down, and then just do it as I was teaching, so I would really just have concrete examples to use and see what the children picked up on and if they were struggling I'd change my examples.

Andy described his approach to examples not directly in terms of abilities of the pupils, but more as being linked to the PNS objectives he is trying to achieve: 'I would say they are important but as long as they meet the criteria and objectives that you're trying to teach, I think that's the main key'. He then relates the sequence of examples to abilities by saying: 'I would order them step by step so start with the simple things, build up so they understand what they're learning'.

An example of this from Andy's lessons is one where the identified objectives are 'to continue to add several 1-digit numbers', 'add several multiples of 10' and 'use known number facts and place value for mental addition and subtraction'. In this lesson the ideas from a previous lesson are extended and developed and the main part of the lesson opens with Andy asking the children to choose six number cards from the range 1-20.

They then select from that subset to match statements such as 'show three cards with a total of 20', 'show three cards with a total greater than 20', 'show two cards with a difference of 5', and so on, each question providing a different example of how the children could match number cards from the given set to a mathematical statement. For two children who Andy considered might not be able to achieve this, an activity with the teacher assistant involved rolling a 9-sided dice to generate a series of strings of single digits to be totaled.

In this lesson, Andy makes use of the strategy of selecting numbers at random for the children to use in their examples, a procedure which Rowland *et al.* (2009) might regard as ineffective since the examples are not chosen for sound pedagogic reasons. It could be argued that the addition of a series of single digit numbers can be rehearsed from a randomly generated list; however, this prevents Andy from setting up pairs of digits within the strings that will add to 10 to make the calculation easier to carry out mentally.

The lesson develops with Andy writing five multiples of 10 on the board and a possible total. The numbers are 20, 70, 80, 40, 70 and the suggested total is 210. Andy asks the children whether the numbers add to more or less than 210 and then asks for their strategies in determining this. After checking that the children have appropriate strategies, Andy then sets them the task of creating examples of addition problems by rolling a dice several times, multiplying the number shown on the dice by 10 and then adding the numbers created.

Further practice of this type of problem is again created through the use of dice, a technique for generating examples which Rowland *et al.* (2009) regard as ineffective for learning, and a good interpretation of Andy's lesson at this point might be to suggest that even using a dice marked from 0-9, the children may end up with examples which are not challenging their understanding of the addition process for combining two 2-digit numbers if, for example, the dice generates all small numbers such as 21 + 32 + 11, where no carrying of units into tens or tens into hundreds is necessary.

In the plenary of one of Victor's lesson, he introduces a set of numbers for the children to use as reinforcement of their learning of addition. He offers the following set of numbers, in the order shown here:

38, 50, 20, 200, 45

A first analysis of this set of numbers might lead to the question of the order in which they are presented. There would be some sense in setting them out in order from largest to smallest or *vice versa*, but the numbers appear to be arranged randomly, once again showing a tendency of Victor to use randomness in his choice of examples, or in this case, the representation of his choice.

The task is to choose any four of the numbers to add, which offers a limited choice to the children, with there being only five combinations possible for choosing four numbers from a list of five. By leaving one number out of the list each time, the permissible calculations can be determined, with the remaining four numbers each time providing the following set of calculations:

38 + 50 + 20 + 200 =

38 + 50 + 20 + 45 =

38 + 50 + 200 + 45 =

38 + 20 + 200 + 45 =

50 + 20 + 200 + 45 =

Each of these calculations presents a different challenge in terms of the make-up of the numbers and the skills needed for the calculation. In the first example, for instance, the order of the numbers presents some difficulty for mental calculation, with the reverse order offering the option to start with the largest number and add on smaller numbers. A similar line of thought can be given to each calculation in turn and this will make some of the calculations easier to deal with than others. If the order of the numbers in the calculation can be varied using the commutative property of addition, children will be able to choose a sequence of additions that suit their personal method preference.

Turning to Naomi's lesson plans for evidence of her approach, she provided a sequence of three lessons on addition which highlight some of her issues in relation to planning and choosing examples, initially for demonstration and teaching purposes but also for providing exercises. In the first lesson, the objectives are: 'To be able to use the expanded method of addition' and 'to be able to estimate to the nearest 100 before adding two 3-digit numbers'. This lesson begins with Naomi presenting a calculation to the children involving the addition of two 3-digit numbers: 437 + 325 = 762

She then asked the children whether the answer seemed reasonable and why. The next example was then used to ask the children to estimate the answer to this calculation: 528 + 247 =

Also, the children were asked whether the answer would be closer to 700 or 800 and why. The children then worked in groups to produce their own 3-digit additions, drawing the numbers from digit cards at random and trying to make an answer as close as possible to a given total. The numbers they had to use were from the set of 1, 2, 3, 4, 6 and 9 and the target total was 500. This offers a limited set of calculations, 120 combinations in total, but many of these will have the same totals, for example, 123 + 469 = 469 + 123, and 123 + 469 = 423 + 169 and so on, giving only 48 different answers, although it might be argues that this should provide sufficient scope to give the children practice at this type of calculation.

The final lesson in this mini-sequence has the objectives: 'To be able to understand that when multiples of 10 go over 100 then they are 'pushed over', 'to be able to estimate to the nearest 100 before adding two 3-digit numbers' and 'to be able to sort different calculations into mental and work out columns'. This lesson continues the theme of the previous two, suggesting continuity and progression, either in Naomi's planning or the unit plans provided by the Hamilton Trust and provides a list of calculation examples for the children to work on, produced in full here:

424 + 378, 424 + 299, 484 + 472, 420 + 320,

524 + 304, 424 + 210, 400 + 324, 424 + 424.

Consideration of these examples demonstrates that they largely fail to be appropriate for the lesson objectives given by Naomi.

The first two examples offer practice at addition with carrying from both units into tens and tens into hundreds, whilst the third example only involves carrying from tens into hundreds (which most closely matches the lesson objective). The remaining five examples are all instances in which the digits in each column total less than 10 and so require no carrying at all. Given the stated objectives for the lesson, these examples from Naomi appear to have been chosen very poorly, either by Naomi's personal choice or by the guidance she has taken from the Hamilton Trust materials or other sources.

### 4.4.5. Representatives of classes

There seemed to be limited evidence of this type of example provided in the data from the case study trainees. This type of example is described as being raw material for inductive mathematical reasoning and whilst it is suspected that primary teachers use such examples, this would more likely have been ascertained through lesson observation.

### 4.4.6 Specific contextual situations

The last of Watson and Mason's (2005) types of examples are those which they describe as examples which might motivate mathematics in a given context. The data collected from the case study trainees in this study did not provide evidence of this type of example, although there would perhaps be opportunities to use investigational situations to generate interesting mathematics, for example, exploring the patterns derived by investigating the arrangement of odd and even numbers in the rows of Pascals's triangle.

### 4.5 Dimensions of Variation

In analyzing the lesson plans and worksheets provided by the case study trainees, it was possible to identify some instances where the trainees have used an example or series of examples in which, with greater awareness of the notions of 'dimensions of variation' (Marton and Tsui, 2004) and 'range of change' and 'range of permissible variation' (Watson and Mason, 2005), could have been developed into better examples from a pedagogical perspective.

Rachael, whilst working with a Year 3 class on adding and subtracting multiples of 10 to various 2-digit numbers, one higher attaining group are asked to use two dice to roll two digits to form a 2-digit number before adding and subtracting 10, 20 and 30 in turn. This generates a set numbers from 11 to 66, with no digits being either a zero or greater than 6. Given the restriction imposed by the dice here, there is clearly the opportunity to extend the range of variation to include not only zero in the digits, but also the digits 7, 8 and 9 to form the numbers for the calculation.

In a later lesson, Rachael offers the higher attaining groups addition and subtraction problems that cross a hundred boundary. One example from her lesson plan for this type of problem is 182 + 22 = . Out of the five digits used in the calculation, three of them are the same (2) and two both appear in the units column of the numbers to be added. This example severely restricts the range of variation and does not extend the children's experiences of adding which could be achieved by other digits.

Naomi gives her Year 4 children the following set of problems, to be solved as if by calculator but with the 5 key broken:

$$42 + 25,\ 42 + 52,\ 64 - 15,\ 64 - 52.$$

It can be seen from these examples that the digit 5 is used in all 4 calculations, presumably because it is the broken 5 key on the calculator that is important. However, the other digits are three 2s and a 1, when a greater variety might have been chosen. This suggest that Naomi may have benefited from an understanding of 'dimensions of variation' and the 'range of permissible variation' as described by Marton and Tsui (2004), expanded upon and developed by Watson and Mason (2005) and reviewed in chapter 2.

Suzy, whilst working on HTU x U with her Year 5 class offers one example which is worthy of inspection in this analysis:

**453 x 2**

| x | 400 | 50 | 3 |
|---|---|---|---|
| 2 | 800 | 100 | 6 |

| Th | H | T | U |
|---|---|---|---|
| | 8 | 0 | 0 |
| | 1 | 0 | 0 |
| | | | 6 |
| | 9 | 0 | 6 |

453 x 2 = <u>906</u>

Here the structure provides scope for a digit in the thousands, yet in the first example there are no thousands which may cause some confusion for the children, although this choice of Suzy's makes broad use of the possible range of variation to include what Watson and Mason (2005) might call a 'boundary example', which helps define the extent of the range of permissible values in such examples.

The second example is 232 x 6 which gives a similar layout to the demonstration example, although repeating the digit 2 in both the hundreds and units denies the children the opportunity to practise a wider range of facts from the 6x table. In this instance, the possible range of variation is not explored as fully as it might be and choosing three different digits for the hundreds, tens and units could have provided such variation in the range of values used in the calculations.

Victor offered his class a set of five numbers (38, 50, 20, 200, 45) from which they had to select four at a time to add. The choice of this set of numbers by Victor is limited and only offers a small subset of all possible additions using 2-digit and 3-digit numbers. Increasing the choice to four numbers from a possible six, if another number was added to the list, would increase the number of combinations from five to 15, giving greater variety and scope for children to work on. On the other hand, it could be interpreted that by offering that set of numbers provided a variety in the types of calculation to be performed, suggesting an approach akin to Marton and Tsui's 'dimensions of variation' (2004), even if unintended as such by Victor, who made no reference to knowing about such a theoretical construct.

Naomi presented a calculation to the children involving the addition of two 3-digit numbers, 528 + 247, then asked the children to estimate the answer to this calculation. It is interesting to note that both this example and an earlier one in the same lesson give an answer in the 700s but both are closer to 800.

This suggests that the choice of examples by Naomi is poor, given the wide range of possible calculations that could have been chosen, relating to the notion of 'range of variation' identified by Watson and Mason (2005) where they discuss the notion of an example space made up of all possible examples in a given context. Naomi's next lesson provided the children with a list of calculation examples to work on, produced in full here:

424 + 378, 424 + 299, 484 + 472, 420 + 320,

524 + 304, 424 + 210, 400 + 324, 424 + 424.

The number 424 appears in several of the calculations, even appearing as both numbers in the final calculation in the list. It appears five times in eight separate calculations, and the digit 4 appears 18 times out of 48 digits used, which seems a high proportion in so few calculations, leading to the interpretation that perhaps that the examples are less randomly chosen than might first appear and that the digit 4 possibly has a personal meaning to Naomi in some way, or whoever wrote the relevant unit plan had a tendency to use the same numbers repeatedly, which again implies that the range of possible variation (Watson and Mason, 2005) has been limited.

A further example is used by Naomi, this being 478 + 284 (=762), which means that for the four examples considered in Naomi's lessons, she had used a number in the 400s three times and one from the 500s once. Given that there is a choice of any digit from 1-9 for the hundreds column, Naomi has limited the range of variation in her examples by only choosing 4 or 5 in many of her examples, and she has not considered using 0 as a hundreds digit, this being interpreted as constituting a boundary example (Watson and Mason, 2005).

## 4.6 Case study evidence on pedagogic considerations in planning mathematics.

### 4.6.1 Higher attaining trainees

The issue of pedagogic considerations for planning mathematics lessons provided some interesting comparisons between the cases studied. The more able trainees, Suzy and Sharon, both discussed planning in terms of checking what was required according to the Primary National Strategy (PNS) (DfES, 2006) and their schools' planning guidance to see which topics needed to be taught during their final placements. They each then gave some indication that they ensured their subject knowledge was at a suitable level for the topics required, revising if necessary, and then set about finding activities and resources from a range of sources to construct series of lessons which would be interactive and creative.

When asked how she began to plan a topic such as multiplication, for her Year 5 class, Suzy described the initial stages as:

> I'd have to check what they already knew, by finding out from the class teacher, and then I'd go into the maths and have to practise it to make sure I know what to do…so I'd sort that out and then…sometimes I go for the starter first, just to try and imagine the lesson I want, or what I'll be doing, and whether you are going to need more time for the teaching point.

She then goes on to describe how she finds ideas for her examples:

> The class teacher basically told me which, erm, some of the sections in the book we could use, so a lot of the time I did just read it through and write up a lot of my own questions for them.

In comparison Sharon, who taught in Year 4, made less use of the PNS and went directly to finding a range of sources for teaching ideas:

> We worked out which topics I was going to do then I went away and planned them, then I sort of looked on the internet at Primary Resources and things to see if there were any fun things… I like to use interactive whiteboard games… which just sort of like get them going. That's the way I plan.

In both cases, there seems to be an underlying assumption that because both trainees are competent in mathematics and achieved high grades throughout the course, they were confident enough not to need to rely on the guidance or content of the PNS and were able to select pedagogically appropriate activities and resources for their teaching.

The overall planning of sequences of lessons and the outline to individual lessons appeared to be drawn from a confidence in the trainees' abilities and subject knowledge levels. When considering in more detail how each of these trainees selected examples for their lessons and particular teaching inputs, there was again a consensus between Suzy and Sharon in terms of approaches to choosing examples. They both talked about starting with easy examples and then making them increasingly more difficult, and also to allocate examples to individual pupils or groups of pupils according to their abilities. For Suzy, the process starts with 'an easy method first', and rather than choosing any examples randomly, she wants to 'definitely make sure it works'. This echoes the findings of Bills and Bills (2005) in practising teachers, that there was a tendency to use simple examples first to ensure everyone could do them and to avoid difficult cognitive issues, in relation to Suzy's understanding and confidence.

However, she goes on to relate her choices of examples to the abilities of the pupils since she is looking to 'make sure there is enough, 'cause with the differentiation, you get one that everybody could actually work through' when constructing sets of examples for herself. When she uses textbook questions, Suzy also provides evidence of careful consideration of which questions to use based on individual pupils' abilities:

> I usually check them through and decide if… 'cause if I was trying to keep in mind all the different children as well, and who could manage what, and then I'd choose those, and maybe a certain group could just do from the book, but just… I'd try to pick out certain questions for them to do rather than just work through it, and then otherwise, write up some… they did end up on a worksheet format, but at least it was more individual.

Their approaches will now be compared with those of trainees whose mathematical abilities can be described as average for their cohorts, to see whether the different abilities of the trainees has any significant bearing on the approaches to planning they use.

### 4.6.2 Middle attaining trainees

This group of trainees showed some similarities in the way they planned mathematics lessons and the way they chose their examples for the lessons, but were all different to the more able trainees. In each case, the mathematically average trainees chose to follow the guidance of the PNS very closely and showed evidence of following more directly the planning guidance they had been given during the university course. In the case of Dawn, she indicated that although she may have wished to plan in a more flexible way, she felt somewhat constrained to follow guidance and as a consequence she felt much safer in so doing.

Dawn further went on to describe an aspect of her experience on placement which she was a little disillusioned with: '…they had one of the set schemes and they followed that very particularly and didn't want anything to, sort of, interfere'.

Rachael seemed to rely totally on the PNS Framework for her planning, taking as a starting point the appropriate 'I can' statement for the topic she was teaching , then developing a three-part mathematics lesson which addressed the differentiation issues within the class. When asked which resources she uses to help her plan the lessons, Rachael replied that she used the Primary Framework. On further questioning about available resources in school, Rachael admitted that there had been a problem with the teachers in the school using a scheme they were not supposed to and that she felt awkward about her planning:

> To be honest with you, it's a bit of a controversial thing, I know that.. … when I got there a lot of people said they weren't really supposed to be doing that 'cause I had asked what kind of planning they did and it kind of got brought up that they shouldn't be doing it and… they had to stop doing it and then they just had to use the Framework.

Andy is also considered to be mathematically average, and his planning approach to mathematics is derived directly from that given in the university course. He describes how he plans for mathematics in a similar way to any lesson:

> I do it very similar to the rest of the way I plan, start with a piece of paper, a broad outline, sort of a ladder, stair thing, like (tutor) taught us, so I know what I'm teaching first and have an overview, then I have a rough plan and then I move on from my rough plan to get a finalized weekly plan. I keep my rough plan, so it's kind of a medium term plan but not, but I have an overview for the 4-week block, erm… I did one week at a time, weekly, and plan that in detail.

This gives little indication of any individual pedagogic reasons for planning in a particular way, and so this was probed further in the next part of the interview by asking Andy how he planned for single mathematics lessons. Again, he explained it purely in terms of taught generic procedures which would apply equally to lessons in any subject area, and not specifically to mathematics:

> Each lesson I have an individual lesson plan and I have key focuses such as what the objectives are, what my aims and outcomes are going to be, or hopefully what the outcomes will be, and what resource I'm going to use, what groupings I'm going to have, who is going to be working with who, staffing wise, basically as much detail in the organization as possible on there, so it's mostly classroom organization.

Looking deeper into this approach to try to get a sense of Andy's approach to mathematics planning, rather than generic planning, he still responded with reference to taught guidance, almost using the PNS Framework as a safety blanket to work from, and giving no sign of his own creativity or desire to experiment with his own approaches:

> From the framework online, it's quite new to me, the new framework, so I found it quite challenging but when you get used to it it's quite straightforward, so I get on there, I work what the blocks are and what block you're going to be in, what letter, so is it A, B or C, and I find it through there, I take my objectives from there.

When discussing with this group of mathematically average trainees the issue of how they choose their examples, they all focus on ability of pupils as the reason behind their choices, rather than any pedagogic consideration from a mathematical perspective. Finally, Dawn was unsure what was meant by examples, but once it had been clarified that they are similar to questions for the pupils, she explained: 'I think you need to, sort of, start simple and progress up to, you know, more complex issues', which resonates with the findings of Bills and Bills (2005).

### 4.6.3 Lower attaining trainees

Turning now to the cases of trainees who are described as being below average in their mathematical ability, both seem to plan their lessons by taking the topic and then drawing on their knowledge to identify activities for teaching. There appears not to be the same reliance on the PNS guidance than in the case of the 'average' trainees, perhaps suggesting that the lower than average trainees do not understand the guidance or do not feel they can attain the levels set by that official guidance, but are content to achieve what they can through their teaching at their own level. Victor, whilst being very enthusiastic about his studies and his teaching, gives an impression of false confidence, but an attitude that suggests he will make it as a teacher whatever it takes. He begins the discussion about planning mathematics lessons by referring to the regular situation of planning a series of lessons: '... in school placement, it's always been from some kind of framework, that the teacher has given us.'

As the study sought to find out more about how regular planning happens, the subject was pursued with Victor regarding how he used the framework he was given, which was the Abacus Evolve scheme:

> So it was a case of just looking at that, and then we had the choice of whether we change or adapt it to suit the needs of the children, or we could just stick with it, to be honest, to suit the needs, 'cause we had a mixed group, mixed age group, er...year group class, it was quite useful to do your own thing so it's...we just used that as a guide basically and then just build around it.

There is a suggestion here that the examples are chosen to reflect children's abilities, adapted from the available resources in schemes such as Abacus Evolve.

Examination of Victor's lesson plans provide evidence of some of the difficulties he has in terms of mathematical ability and knowledge, and how these are reflected in his teaching. In one particular lesson, Victor records the objective as:

> Children are to learn how to attract [sic] and subtract multiples of 10 from 2-digit numbers. They are to attract and subtract 2-digit numbers using skills they have learned about multiples.

The thing to notice about Victor's lesson objective is the choice of vocabulary which is as stated on the lesson plan. It is therefore possible that he will use incorrect vocabulary with the children he teaches which will inhibit their mathematical learning and understanding, using 'attract' rather than 'add'. As seems to occur often in Victor's lessons, a series of randomly generated examples are provided, which are not regarded by Rowland *et al.* (2009) as good examples due to the random nature of the numbers generated and the inability of the teacher to control the variables sufficiently to ensure pedagogic understanding, a feature which occurs across a number of the cases in this study, but particularly in Victor who confesses to his own lack of self-confidence in mathematics.

Naomi's approach mirrors Victor's in terms of identifying the required topic from a framework, in her case, the Hamilton Trust materials, then identifying the teaching activities and resources that will be needed, drawn from her own knowledge and related to that of her pupils. When asked how Naomi would use the Trust or Framework materials to develop her plans, she responded by saying:

> Well I'd read it first, see what part of maths I'm teaching and I'd probably think, I can do this, and probably do a cloudburst with different ideas, obviously I'd have to know what the ability of the children are, and that's what would probably influence me, I'd probably have lots of different little activities planned because I wouldn't know what the children can do.

## 4.7 Perceptions of Subject Knowledge and its Impact on Choice of Examples

This section will focus on the final research question which was: 'Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?' In analysing the trainees' lesson plans and interview transcripts, evidence was sought and found for links between mathematical subject knowledge and how it impacted on trainees' choices of examples. This also gives an indication of the perceptions of the case study trainees in terms of how importantly they regard examples and how significantly their choice of examples is influenced by their level of mathematical subject knowledge. It became apparent from the interview transcripts that the case study trainees had largely misinterpreted the intentions of some of the interview questions, and in their responses showed a lack of awareness and understanding about the nature and purpose of examples.

Both the trainees in the higher attainment group had studied mathematics to A-level, and this seemed to have affected each of them in regard to their approach to planning and teaching primary mathematics. Suzy spoke of her awareness that her ability should have a positive impact on her teaching:

> I should be better than I am because I did 'AS' level, but then it's still different... the methods of laying it out has changed, because the way I learnt it, and now I'm having to re-learn it.

She describes how she revises for topics by re-visiting her GCSE books and also suggested there might be a link between subject knowledge and choice of examples by stating: 'I think if you understand the thinking behind each question, then you'd know... which ones are better'.

However, Suzy was not clear what was meant by examples, she took this to mean which questions she would ask. When she was asked to explain how she would choose her examples, or questions, for her lessons, she responded:

> Sometimes I started with an easy method first, and then go through it on the board, so I model it, then I got them to write it down in their book because a lot of them struggled with the concept, they were also...if everyone was doing it, and then if you were recapping I would get one of the children to come up and help do it.

This suggests Suzy focuses more on asking questions to engage the children with procedures, rather than offering them examples in different contexts which might lead them to understand a concept in greater depth.

Sharon had achieved A-level in mathematics, and noticed how her detailed knowledge of mathematical topics and their procedures had shifted from studying A-level back to focusing on primary mathematics:

> I know it's something I found harder at first, especially going back to the real basics, because you do forget about them, 'cause you start doing silly things like integration and then you can't go back to just doing the simple things so now after like three years of going back to it, it's okay because I know what to do now, and I wouldn't probably be able to go back to integration again.

Further, Sharon considers that her choice of examples is usually pitched too high for the children as a result of her own level of mathematical knowledge. She spoke of 'skipping out the easy bits' and the 'need to dumb it down a little bit for children' in relation to her examples and explanations. When asked about the importance of the examples used in children's mathematical learning, Sharon responded: 'I think...yeah, it's really, really important.'

She went on to explain how she wanted the children in one lesson to subtract a range of 2-digit numbers from 99, the choice of minuend being significant in allowing the children to make the subtraction in columns with no need to 'borrow'. Sharon also showed evidence of a mixed approach to planning her examples, suggesting that sometimes she might choose examples randomly, whereas other times she decides which examples will be best for the learning she is trying to achieve for her pupils:

> 'I don't necessarily plan exactly what numbers I'm going to use, but, like, for example, if it's multiplying and I knew I was going to multiply it by 4 then I deliberately wouldn't pick difficult numbers because it just gets too hard for them, but with the angles I knew which ones I wanted them to do and knew I wanted them to start with 180 and 90 and if they could do that, give them a go at doing a 45 degree angle because that's more difficult than the other two.'

Sharon offered evidence through her interview that there is a link between subject knowledge and choice of examples, although she seemed to believe that being a higher attainer in mathematics could be both an advantage and disadvantage at times:

> I think I have to be more aware of what I choose because I would probably pitch it too high every time, and I know that that was something I kept thinking of and being really aware of, don't pitch it too high because you are used to working with higher attainers as well, and I think it helped me think about my examples a lot more whereas someone who struggles with maths a little bit may…really thoroughly plan a maths lesson and really think about those things … it was okay because I had the basic knowledge there, and I could help them work through the problems, but someone who did struggle with maths, they would probably want to read through it and recap it…so it sort of works to and against you, I think….if you're better at maths then you should be better at teaching maths, but it's different understanding it and teaching it, to understand it you just have to grasp concepts, but to teach it you have to be able to explain how you got it which is a completely different process.

Amongst the trainees in the middle attaining group, both Andy and Dawn remarked on how they feel confident about their levels of mathematical knowledge, in Andy's case he felt more confident after three years of the course, but admitted he still goes away to revise topics before teaching them, particularly those he has not taught before. For example, regarding one topic, Andy describes how he has '…to go away and do a bit, there's so many things in fractions'. Dawn acknowledged that her mathematical knowledge was not as good as she had thought when she started the course, but after 20 years of using mathematics in everyday contexts, she was able to improve quite quickly and feels confident that the course has given her the level of knowledge she needs to teach effectively and choose good examples, although she did need to ask what was meant by an example before offering:

> I hadn't used maths apart from like, the everyday maths that you use as an adult in terms of sort of, working out your family budget… I'm married to a person who measures how much paint we need for the kitchen and how many tiles we need… so I've not done that… it was actually much better than I thought it was, even things like algebra, which is probably my weakest subject, is okay.

Only Rachael from the middle attaining group gave a firm impression that she was not very confident about her mathematical knowledge. She describes her struggle to achieve GCSE saying: 'When I was at school I never really enjoyed maths. I had extra tutoring from a maths person, it was more a confidence thing… that really helped'. This lack of confidence and perhaps knowledge seems to be still causing Rachael a problem in her placements: 'When I was with Year 6 I asked the teacher a lot, how do we do this… 'cause when I got to Year 6 there were some things that I wasn't really sure about.'

Rachael also suggested that she felt examples were not necessarily the most important feature of her mathematics lesson, rather it is the lesson itself with its objectives and resources:

> I think examples are really important but I wouldn't necessarily consider theory when I was choosing the examples, but I might do for when I do my planning, but not when I was putting examples on the board, I think I'd be more just making them up in my head.

She regarded mathematical subject knowledge as being significant in the ability to choose effective examples for learning:

> I reckon if you're more aware of the subject knowledge and how to teach it, and if you've got that step ahead, then you can choose examples. A good example would be one where they learn quickly what it is you are trying to teach them.

Amongst this group of middle attainers, all three trainees were sure that having better subject knowledge would enable better examples to be chosen. Dawn expressed this by suggesting that 'at the moment it's a case, I think, of you know, sort of working out what you need to get by, I would like to have a sort of greater depth of knowledge'. Rachael was more positive about the link, saying: 'if you're more aware of the subject knowledge and how to teach it, and if you've got that step ahead, then you can choose better examples'.

Turning to the two lower attaining trainees, Victor admitted that he 'always struggled with maths', but that he enjoys teaching it and spends time reading and researching the topics he needs to teach. He acknowledged that his own lack of mathematical knowledge has a limiting effect on his teaching but he did, however, appreciate and agree with the idea that good knowledge leads to better examples:

> I think if you're confident in something, you know a lot about something... and if a child poses you a question... and you can explain, then that's a real strong point I think.

Naomi had a view which was different to all the other case study trainees on this key question. She described how the examples needed to be chosen to suit the class and that subject knowledge did not necessarily play a part, rather good teaching was the issue. She was keen to point out that recalling good examples from how she was taught could be helpful, but that on the whole, it is the teacher's ability to know the class that is crucial:

> You are always going to have a memory that sticks out, either an example or how one of your secondary school teachers taught you something, and then you can obviously apply that to your lesson, but sometimes I think it's just good teaching to choose a good example to suit your class, I think you've got to know your class to choose a good example.

This view was then followed up by remarking on the difference between trainees who had reached higher levels of mathematics and those who had only achieved GCSE:

> I think someone who got an A at GCSE or an A-level is going to baffle the children because they're only Year 2, their concept of... their maths skills are very limited.

It is likely that this comment is based on Naomi's low perception of her own ability, but she clearly felt that despite her limitations in subject knowledge, she was better equipped to choose examples well because she would not confuse the children and because she knew the children's capabilities.

### 4.8 Summary

This chapter has explored the data collected from trainee primary teachers against the research questions set out in Chapter 1. The key themes which emerged from the data were in relation to how trainees plan their mathematics lessons, how they choose example and the extent to which subject knowledge impacts on their planning and choices. Supportive evidence from the data sources including interviews and lesson plans from a number of case studies showed the range of approaches used by trainees.

By examining the data against the theoretical frameworks of the Knowledge Quartet (Rowland *et al.*, 2009) and Watson and Mason's (2005) typology of examples, it has been possible to identify ways in which trainees perceive, select and use examples in their mathematics teaching. This chapter has shown that there a number of broad features about how trainee primary teachers work with mathematical examples, related to their levels of mathematical subject knowledge and particularly in terms of choosing examples to match the different attainment levels of their pupils. These outcomes will be drawn together in making conclusions about the study and recommendations for teacher education practice and future research in the final chapter.

# Chapter 5 Conclusions

## 5.1 Introduction

In this chapter the original objective for the study is revisited, to examine pedagogic considerations when primary teacher trainees choose mathematical examples and how the trainees' mathematical subject knowledge impacts on their choice. This will lead to a reconsideration of the research questions that were intended to be answered by the study. The main findings from the research will be reviewed and summarized and the extent to which the research questions were answered will be addressed. The outcomes of the study will then be discussed in the light of existing research and there will be an indication of how the findings integrate with, expand on, or contrast with previous research.

There will be a brief discussion of aspects of the literature review that seemed most pertinent to the pedagogic considerations and subject knowledge implications for the ways trainees choose mathematical examples. The findings of this study will be considered in the context of the Knowledge Quartet (Rowland *et al.*, 2009) and the work of Watson and Mason (2005) and finally there will be a consideration of suggestions for possible future research and implications for practice in primary teacher education and the training for those entering the profession.

The study provided some significant findings, which are listed here and explored in more depth later in the chapter. Firstly the relations which appeared to some extent with the case study trainees between their potential as mathematics teachers and the way they approached example choice was revealed in the interview responses. Secondly, trainees' lack of understanding about examples was clearly evidenced from the interview, and thirdly there was strong evidence to suggest that the discourse of pupil "ability" and the related concern for differentiation influenced trainees' choices of examples and other learning opportunities for their mathematics lessons. Each of these significant findings is explored in greater detail shortly in response to answering the three research questions which underpinned the study.Throughout this study the debates about mathematics subject knowledge have been constant reminders about the challenges facing primary trainee teachers, who are largely not mathematics specialists but who nevertheless need to attain a level of knowledge and competency to ensure their teaching is effective.

The Knowledge Quartet of Rowland *et al.* (2009) has been an extremely useful framework against which to evaluate and analyse data from trainees' lesson plans and interviews, and the identification of pedagogic approaches and subject knowledge impact amongst a set of case studies was enhanced by comparing the examples they chose with the types of examples identified by Watson and Mason (2005). It was from an awareness of the range of examples, both good and bad, that trainees choose in their teaching, that inspired this study and the outcomes suggest there is little in the way of deep pedagogical consideration amongst trainees of all attainment levels in mathematics.

## 5.2 The Research Questions

The initial objectives of this study were:

1. To find out which pedagogic considerations a cohort of trainee primary teachers use when choosing mathematical examples in the classroom.
2. To establish where these pedagogic considerations fit within current theoretical frameworks in primary mathematics education.
3. To investigate the relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select.

Based on these objectives, three research questions were drawn up to form a basis and focus for the research that followed, directly linked to the objectives:

1. What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?
2. How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?
3. Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?

In order to examine the implications of these questions, recent and relevant literature was searched to form a theoretical basis for the study. The starting point for this was the work of Shulman (1986) in which types of knowledge for teaching were identified. Out of these, two in particular were linked to mathematics teaching by Rowland *et al.* (2009) which were subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Debate about how these forms of subject knowledge arose in the late 1990s when Initial Teaching Training providers were subjected to a 'curriculum' of mathematics coverage for their teacher training courses, and as a result research into subject knowledge, its forms, purposes and measurement came about.

This study arose out of a background of primary teaching, mathematical enjoyment and expertise and a period of teaching in Initial Teacher Training. Concerns grew about the way primary trainees who were not mathematics specialists, or even particularly competent at school mathematics, were planning to teach in school placements and choosing examples for learning. It was suspected that the planning process, whilst following government and university guidelines, was rather more bureaucratic than pedagogic and that children were being offered mathematics activities and examples which would not engage them in mathematical thinking or challenge their learning in order that their learning might develop appropriately.

1. What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?

This question was designed to seek to identify the reasoning behind trainee primary teachers' planning for mathematics in general and their choices of examples in particular. It was answered in part by the responses the case study trainees gave in their interviews, and identified through analysis of the resulting transcripts. Examination of, and some discussion about, their lesson plans highlighted some of the examples they chose which did not always seem to match their stated lesson objectives. Across the seven case study trainees there was a variety of approaches which can broadly be described as either 'reliant on Primary National Strategy', 'reliant on other sources' or 'own knowledge'. Each of these approaches was evidenced from both the lesson plans and interviews.

The trainees who relied on the PNS came from all levels of attainment. Sharon, a more able trainee, described how she always checked the PNS to make sure her objectives were 'right', and then she looked for resources that would enable her to teach to the objective. However, Sharon felt she had sufficient confidence not to have to rely on the PNS if she was in a school that permitted her not to. All three of the middle ability trainees referred to using the PNS as their guide when planning, for example, Andy planned weekly sets of lessons based on the blocks and units of the PNS. He supported the plans with materials from other sources such as the 'Primary Resources' website, or '100 Numeracy Lessons', as well as using sections from the Abacus Evolve scheme, as long as they matched the PNS objectives.

Rachael based her lessons around the PNS framework and its 'I can' statements, an approach which is promoted by the university, as she felt this gave her the guidance she needed to ensure a 3-part lesson with appropriate differentiation. Dawn always checked her methods against the PNS to ensure she was teaching the way it suggested, but she had a more flexible approach in that she tried to link different elements from the PNS into a 'wallmap' of connected ideas. Finally, from the lower attaining trainees, Naomi based most of her final placement plans on the Hamilton Trust materials, since that was the school's preferred method, but she also tried to ensure that the objectives matched the PNS 'I can' statements and chose tasks from the PNS to use in her lessons.

The only trainee amongst the seven case studies who did not use the PNS but relied on other resources was Victor, one of the lower attaining trainees. He admitted to having what he described as a 'personal stock of ideas', but also was guided by the school in how to plan. He relied mostly on the school's adopted scheme, Abacus Evolve, which he either used as it stood, or he made alterations and modifications to suit either the objective he was teaching, or to suit the abilities of the children so that the learning was differentiated appropriately for different groups of children.

Finally, one of the more able trainees, Suzy, described her approach as being a confident one which relied on her own level of subject knowledge, which she considered as good because she had studied to AS level in mathematics. She spoke of having consulted textbooks to check for methods or activity ideas, but always wrote her own examples, either from her own knowledge or by adapting ideas for examples from books.

Across the seven trainees, there were two approaches to preparing sequences for examples for children to work on. The most common approach was to provide graded examples for the children, starting with what trainees regarded as easy examples and moving through more difficult examples until reaching the most difficult ones. It was felt by each of the trainees who used this method that children would then be able to work according to their attainment, with more able children completing more of the examples or perhaps starting at a point beyond the simplest examples, whilst lower attaining children would start from the easiest examples and work to a point where the examples became too difficult for them.

The alternative approach to this was demonstrated by three of the trainees, who each described how they produced different sets of examples for different groups of children, based on their understanding of each pupil's attainment. This meant that each child had a set of examples that they could complete at their own level, although within the differentiated sets of examples, more than one trainee described how they did not necessarily move from easier to harder examples, but often mixed up the difficulties of the examples at random to ensure that the children within the ability group had sufficient and regular challenge.

In summary for this research question, the evidence seems to suggest that whilst all the trainees are made aware of the Primary National Strategy as part of their training, the extent to which they use it once in school varies from using is as a major support and guidance for their planning, however, a minority of trainees either refer to it as useful guidance but not exclusively, preferring to draw from a range of other resources or their levels of subject knowledge where they are confident that these are suitably high.

However, perhaps the over-riding feature demonstrated by the research data in relation to this question is that the seven case study trainees, from a range of mathematical attainments and backgrounds, were not at all clear about what constitutes a mathematical 'example'. Therefore, any discussion about their choice of examples for mathematical learning was limited to their interpretation of examples, which was usually incorrect.

2. How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?

This question was rooted in the notion that whilst trainees on a teacher training course would be presented with, and expected to engage with at a critical level, a range of theoretical approaches to teaching mathematics effectively, either in texts or academic articles, it would be of interest to try to determine the impact of such theory on their classroom practice. If there was little evidence of trainees using the theoretical ideas they had been exposed to in their training, then the relevance of including such theories in the training could be questioned, either in terms of their inclusion as such, or the way in which trainees interact with the theory for the purpose of assignments, rather than drawing specifically on the theory for their teaching practice.

The question was approached through collecting data from interviews with 25 trainees from two cohorts over two years, and the responses of the selected sample group of case study trainees were examined in detail for evidence of how they used theoretical approaches in their teaching. The question was answered through a combination of analysing the case study trainees' interview transcripts and triangulating this with the evidence from the lesson plans they provided, along with supporting documents in the form of worksheets of examples.

It appears from the range of evidence collected in the data that for the case study trainees in this study that whilst they were able to recall engaging with aspects of theory for various module assignments, they were collectively rather inconsistent in considering any theories specifically when planning mathematics.

Out of the seven case study trainees, four of them denied having heard of any theoretical models or frameworks, whilst the other trainees had very limited recall and in two cases, only when prompted. Suzy, when asked if she had come across any theories that might have guided her in choosing examples replied: 'Not any that I can think of', and Dawn, in answering the same question replied: 'I don't think so!' Both of the trainees who were regarded as lower attainers, Victor and Naomi were unsure that they had encountered any theories. Victor answered by saying: 'Erm... not really, no... not that I can remember... it doesn't really come to the fore in my planning, anyway, when I'm in school'. Naomi did not think she had used any theories, although when prompted to see if she was aware of the ELPS theory (Liebeck, 1990), she asked to be reminded what it was, described how she had only used it explicitly in essays then related her thoughts about it as:

> I know what it is, yeah, I just forgot what it stood for. No, I've never used it, I suppose subconsciously I probably use it in some of my lessons, but I've never consciously thought I'm going to use ELPS for this because I think it's for me, personally, it's dependent on the children as well or as much as the theory behind it.

The remaining case study trainees made reference to using the ELPS model of Liebeck (1990), Andy was particularly positive about it in his interview:

> ELPS, yeah! The big one! An awful lot of ELPS. It didn't used to (help with planning) but yes, definitely now... what resources are going to be visual, what resources I'm going to use, what language I will use, like, what mathematical words am I using... I wouldn't say I use the word ELPS but I use that idea.

Rachael firstly recalled elements from the Primary National Strategy framework as *'theory'*, such as the 6 Rs, such as recall, rehearse and so on. She then commented on using the ideas of VAK (Visual, Auditory, Kinaesthetic) in all her planning, 'especially this time I was trying to do visual all the time for the EAL children', and she also described using ELPS, particularly in relation to making the learning experiential, such as setting up a shop when the children were learning to use money and calculate prices and change.

Finally on the awareness of theories are the views of Sharon, a more able mathematician, who began to relate theories to teaching ideas, drawing on her knowledge of textbook authors who have written teachers' books:

> There's quite a few good books, 'cause I did my dissertation on maths, so there's quite a few good books... but a lot of them, I know, is it Cockburn, she has quite a good book for examples on subtraction... it gives you confidence that you know you've sort of dealt with them... and Chinn's got a few good ideas as well.

After prompting her to think of academic literature she may have read, Sharon then talked about Liebeck (1990) but described the role of academic theories based on her experiences in school:

> While I'm at uni, yes, but not necessarily when in school because no one introduces you to these people, like I only knew about Liebeck because we were taught it through maths, and through the idea of actually following it through and it's something you'd probably do but without knowing that it's related to a person's theory, and a lot of teachers don't know about these people because no one comes in and no one tells you about them and tells you who to read about.

Sharon seems to have a view that teachers already in school have not experienced the theoretical input she herself has had during her course, perhaps not thinking that the teachers all went through their own courses at some time.

In summary on this research question, the evidence from the data collected points to the likely interpretation that trainee primary teachers are aware of theoretical frameworks whilst completing their course, but do not necessarily connect the theories to classroom practice and do not, on the whole, use theoretical notions to aid their planning or choices of examples.

3. Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?

This question was designed to consider the links, if any exist, between trainees' levels of mathematical subject knowledge and how their level of knowledge might relate to the choices of examples they make. It was answered to some extent by considering the range of data about each trainee obtained from pre-course and in-course results of tests and examinations including GCSE and GCE results, mathematics tests at interview and mathematics diagnostic test results from early in the course. The mathematics module results were also taken into account since these give a measure of trainees' knowledge and understanding in relation to subject knowledge and understanding of the teaching and learning of mathematics, along with planning for lessons.

Drawing on the evidence from the case study trainees' lesson plans and interview transcripts, it can be interpreted that whilst each of the trainees identifies with the idea that subject knowledge is related to the choice of examples, their views on what subject knowledge includes and their understanding of 'examples' leads to a blurred interpretation of that relationship, which links with the findings for the first research question.

Dealing with the trainees in ability groupings, the more able mathematicians had both reached AS level, with one of them going on to complete A-level. They each related how their understanding of how to teach primary mathematics had been influenced by studying mathematics beyond the course requirement of GCSE grade C, Suzy described how she thought methods for calculations had changed and that she had to revise her own strategies, whilst Sharon had forgotten the basics while she learned advanced topics like integration, but now she was engaged with primary teaching, she had easily forgotten the advanced work. Sharon also felt that her higher level of study in mathematics had led to her often pitching examples too high for children and needing to simplify them or explain in different ways to help the children understand. Suzy, however, was firmly of the belief that higher levels of study and the greater understanding that it implies would certainly lead to her being able to choose better examples for children.

The group of trainees identified as middle attainers expressed differing levels of competence in their own mathematical subject knowledge but all agreed that greater levels and depth of subject knowledge leads to better examples being chosen for lessons. Dawn had achieved mathematics O-level over 20 years previously and was very confident in her mathematics. She also claimed to think more carefully about her examples to avoid confusing children. Andy felt his confidence had improved greatly after the 3-year course, but he still revised thoroughly before teaching a topic to ensure he could cover any misconceptions the children might have.

Rachael was the least confident of this group, she confessed to having had extra mathematics tuition to help her achieve her grade C for GCSE and whilst she had personal concerns about her perception that different schools do things differently, she believed that greater subject knowledge allows the teacher to be a step ahead of the children and therefore choose examples that will benefit the children.

The two trainees who were identified as being least able mathematically, Victor and Naomi, seemed to have similar levels of attainment with their school level subject knowledge and exam results, but the way they approached their teaching seemed to demonstrate different levels of confidence. Victor admitted to struggling at school in mathematics, and for his teaching placements, he needed to revise and research topics before attempting to teach them. He also confessed, as described in chapter 4, to sometimes leaving out parts of the planned mathematics teaching if it was something he felt unsure about.

He was not confident to suggest whether better subject knowledge led to better examples, but simply referred to his perceived low level of knowledge and the fact that he felt he needed more guidance when planning and teaching mathematics. For a trainee at the end of the course and on the verge of taking a full-time teaching post, this was a piece of evidence that may concern school leaders seeking to employ Victor and it might be hoped that he would continue to seek advice on how to choose examples carefully and work on how to teach effectively with them.

Naomi felt she had achieved the best she could at GCSE, making the required C grade, but realised that she needed to continue to work on her knowledge and understanding, particularly when teaching more able children. In order to develop her knowledge, she described how she continued to use BBC Bitesize as a resource for revision. With regard to the link between subject knowledge and choice of examples, Naomi was the only trainee in the sample case study group who suggested that it was the teacher's knowledge of the class and their abilities that helped determine which examples should be chosen and not necessarily the teacher's level of mathematical subject knowledge or any relation to a typography of examples.

**5.3 Comparison of cases**

In selecting seven trainees to examine in depth as case studies, it was expected that there would be similarities and differences between the trainees. These would, it was thought, be related to their mathematical attainment prior to starting the teacher training course, and the also in some way to their success or otherwise on the course. For these reasons, the trainees were grouped into higher, middle and lower attainers for much of the analysis of both lesson plans and interview transcripts. Looking across the cases, an assumption was made that the attainment groupings would fit closely to any observed similarities and differences in terms of how the trainees planned mathematics lesson, chose examples and made use of theoretical frameworks.

After the data was collected, sorted and analysed, as described in chapters 3 and 4, it became apparent that some of the distinctions between the cases could not simply be explained by the levels of attainment that were identified. One example where this is clearly the case is in terms of the theoretical frameworks that trainees used to support their planning.

In the case of the lower attainers, Victor and Naomi, they were both unaware of any particular frameworks when first asked, although on prompting Naomi remembered some aspects of these. For the middle attainers, Andy identified ELPS and VAK as significant, with reference to writers such as Haylock and Hayes, Rachael only recalled the ELPS and VAK models with prompting, whilst Dawn firmly stated that she had no recollection of any theories or models from her course. With the two higher attainers, Sharon recalled the ELPS model as well as work by Cockburn and Chinn, but Suzy denied knowledge of any theories that might help with mathematics planning, teaching or learning. So for that aspect alone, the ability groupings appeared to have no direct relationship on trainees' use of theoretical frameworks.

Another comparison that can be made across and between the cases is with regards to their perception of the link between subject knowledge and choice of examples. All the trainees in the higher and middle attainers agreed that better subject knowledge and understanding would lead to better examples being chosen. However, neither of the lower attainers put forward that idea, one preferring to suggest that the teacher's knowledge of the children led to better examples and the other pointing to his own need for further help with subject knowledge.

Looking back through the interview data, it is also apparent that none of the trainees who were in the case study group mentioned the importance of assessment in choosing examples, even though assessment can be regarded as a theoretical perspective. It may have been interesting to explore this aspect, relating trainees' choices to their assessments of children's prior learning.

## 5.4 Limitations of the research

The study has attempted to identify factors that influence how trainee primary teachers plan for teaching mathematics and choose examples for learning, and the data collected has enabled some broad conclusions to be made. However, the study has limitations with regards to its aims, sample and data collection which will be summarised here, followed by recommendations for future research which could alleviate some of the limitations described.

The first issue is the scale of the research, being located in one university, drawing on trainee teachers' cohorts from one B.Ed course. This will limit the outcomes in the sense that all the trainees under scrutiny for the study were taught the same programme by the same small group of tutors. This is likely to have influenced the trainees' thinking and practice according to the advice and guidance they were given during the course. The choice of final year trainees was made to provide a balance between sufficient course coverage and inexperience in the classroom, but this in itself will provide a definitive sample from which to choose and may exclude some trainees from other years who may be able to offer broader and deeper insights into the factors that affect planning and choice of examples.

Secondly, the range of data collected was limited. The quantity of lesson plans collected gave a broad range of possible topics and examples to examine and analyse, but for reasons of focus, these were narrowed to the year groups and mathematical topics previously described in Chapter 3. The interviews with a number of trainees provided a range of material for qualitative analysis, but only a small number of the interviews were chosen to focus on as case studies, depending on the year group of the trainees' placements, the topics they were teaching and also their willingness to be interviewed and submit lesson plans. The case study trainees were selected from those who were from the most commonly represented year group, taught the most common topics, supplied a range of suitable lesson plans and were willing to be interviewed.

At one stage in the planning of the research, it was proposed that some lessons were videoed for subsequent analysis but this was not possible as a result of student assessment issues within the university where the research was set. However, this gave the opportunity for this study to bring new kinds of data to the study of trainees' choices of examples, with the use of lesson plan analysis and semi-structured interviews offering a different perspective to previous studies in which data was mostly observational. The use of interviews gave the trainees the opportunity to reflect on their teaching and their choices of materials in a way which is not often possible in such depth when discussing a lesson observation with a university tutor.

### 5.5 Implications for Future Practice

As a practitioner-researcher, I feel it is important to reflect on the research that has been carried out and make some comments about my role as a researcher throughout the study and how this might impact on my future practice as a researcher and mathematics educator. The act of reflecting is a complex one, and to be useful it needs to do more than look back and celebrate the good points or lament the bad points. In terms of this study, I need to consider how the outcomes will affect what I do in mathematics education in future, drawing on the experiences and actions of this study and 'pro-flecting' (Mason, 2008), that is, trying to imagine how I will use what I have learnt and acting in a different and hopefully better, more effective way in teaching and learning situations that might arise.

This study arose out of my background and experiences as a learner and then teacher of mathematics. Both aspects continue to be a significant part of my life and this study has brought out a number of developments in my multi-faceted role. As a researcher, I have been able to explore the depths of postgraduate research, finding that having an interest in the research topic is vital, but being aware that other elements come into play during the research process. I became aware to a significant level during the research that there was far more than a simple relationship between my desire to find out and the way of doing this.

The research experience raised my awareness of how, as a researcher, I brought to the study my own thoughts, aspirations and feelings, as well as 'who I am', or my identity in terms of race, gender, family background, schooling and so on. These factors can be both implicit and explicit in the research and knowing when to repress or enhance different features to help in the construction of meanings throughout the research project was an essential development in my researcher awareness. Also amongst the elements of doing research are drudgery and frustration, at various times of the research design, the data collection and its analysis, and whilst I understand the need for rigorous assessment of the standard of the finished research study, motivation to revisit sections for post-viva revisions was very hard to find. However, I feel that having reached the end of this particular research journey, I have emerged a changed person, a researcher with more effective research skills and an academic with deeper knowledge about an aspect of my field of mathematics education.

At a practical level, this study has implications for myself and for the wider mathematics education community. For my own position, completion of this research will enable me to find outlets for dissemination. These include within my own institution, regionally and nationally through workshops, conferences and informal discussions with other mathematics educators. In my role on the Teaching Committee of the Mathematical Association and also its Primary Subcommittee, a joint group with the Association of Teachers of Mathematics, I will be able to offer more effective input to discussions and documents and contribute to consultations on national developments, for example the new National Curriculum, as well as raise awareness about example choice.

There is also scope to use the findings as part of school-based work which is carried out on a consultancy basis, providing school INSET and regional teachers' courses. Currently there are very limited opportunities to be directly involved with ITT provision at my institution, although I have regular meetings with trainees as part of my role in Academic Writing and Numeracy support, and I anticipate that these opportunities may increase with time. It will be useful to make opportunities to discuss my findings with the ITT staff.

In terms of how the outcomes of this study might inform future practice at a local, national or international level, there are a number of issues which could be considered and implemented if pursued through the appropriate channels. These issues are presented now as recommendations, listed in relation to the research questions.

The first research question focussed on trainees' pedagogic considerations when choosing mathematical examples, and based on the outcomes of this study, where many trainees seemed to feel they should plan mathematics based on attainment groupings, I would recommend that trainees are made aware of the possibilities of using mixed attainment teaching and learning in mathematics. Whilst such an approach was not part of the planned research in this study, finding that pupil attainment was one of the significant issues in trainees' approaches would suggest that development of awareness of the benefits of mixed attainment teaching could enhance trainees' choice of examples in future, and the ITT staff within the institution should give this some consideration.

If trainees become aware of different ways of teaching according to pupil attainment, then the planning approach itself may be altered significantly and for the better. With the demise of the Primary National Strategy in the United Kingdom and the development and implementation of a new National Curriculum from 2013, there is scope to begin to teach trainees how to plan effectively for mathematics around the new curriculum, making use of more creative approaches and mixed attainment groupings, and I would recommend an overhaul of the current planning approaches currently used within the institution for this study and possibly further afield.

The second research question was focussed on trainees' use of theory in planning mathematics. The findings suggested that theory was only seen to be relevant for academic study and was therefore used predominantly in module assignments and not in classroom practice. I would therefore recommend that theoretical frameworks were used that had a clear and direct impact on classroom practice, which could also be used to analyse that practice in the academic aspects of trainees' courses. The current framework of choice in the institution of this study is that of Liebeck (1990), whose ELPS model has been taught for a number of years. I would recommend, on the basis of this study, that the Knowledge Quartet of Rowland *et al.* (2009) would be a more useful theoretical model to adopt, given its strong research basis and its direct application to mathematics teaching and learning. Such a model could then be used as a way of helping trainees think about their planning and teaching of mathematics, as well as for reflecting on the teaching and learning.

The third research question tried to examine the link between subject knowledge and choice of examples. Whilst there were some differences between trainees of different mathematical attainment, the overall finding was that in this study, trainees could see that better subject knowledge in mathematics would help them choose better examples. I would therefore recommend that mathematics modules increased their focus on subject knowledge in the first instance, perhaps replacing some of the more generic lectures and workshops on aspects of professional practice such as planning, resources, assessment and so on, with more in-depth sessions that teach trainees the mathematics they need to teach effectively, developing their SMK and PCK to a higher level. This could also include, as a final recommendation, a greater awareness about mathematical examples – their nature, types and uses in teaching and learning.

## 5.6 Recommendations for further research

In the light of the limitations described in the previous section and the outcomes discussed in chapter 4, this study raises a number of issues which would be suitable for further research in this field. Firstly will be outlined some approaches which would specifically address the limitations mentioned previously.

The first limitation was with regard to the setting of the research being in a single university. This could be addressed by extending the research to consider how trainees from a range of universities approach their mathematics planning and choice of examples.

This would involve variations in geographic locations and potential localised cultures of practice and the influence on cohorts of trainees by different mathematics education tutors.

Further to this, a natural extension to the research would be to consider trainees at different stages of their training. This could involve first and second year trainees from B.Ed programmes, or trainees from PGCE programmes, both single and two year course. Beyond this, research into how serving teachers approach their planning and choice of examples would add another dimension to any findings. This could be further divided into Newly Qualified Teachers, teachers with up to five years experience, five to ten years experience and so on. The distinction between mathematics specialists and subject leaders could provide an insight into how schools approach the planning process and this could be a fruitful source of investigation.

Another limitation described in the research design was concerning the topics and year groups used for sampling lesson plans and examples for learning. Further research would be useful in looking at the full range of topics in mathematics, covering all attainment targets of the National Curriculum, which would encompass aspects of shape and space, measures and data handling. The full range of primary year groups could be researched to examine and analyse planning and choice of examples, which could provide useful data about how teachers who specialise in a particular Key Stage or Year group approach their planning of mathematics.

Looking beyond the primary phase, further work of the type undertaken in this research could focus on Early Years mathematical experiences, or Key Stages 3, 4 or 5. Potentially, there is scope to examine the research questions of this study for those who teach mathematics in Further and Higher Education.

In order to extend this study, one of the most beneficial approaches would be to combine data collection of lesson plans and interviews with other data such as video-taping of lessons, interviews before and after the lessons with the trainees, and interviews with other participants in the lessons, such as teaching assistants and the children. The use of video tapes from lessons can be analysed in their own right, but could also be used in video-stimulated reflection with the trainees to enable them to focus on aspects of their practice which might improve when they see their teaching from the perspective of observer.

One issue which began to surface in this study was that of choice of language with examples, but there was not sufficient scope to include within the research design. Trainees were heard to use language during the interviews that suggested three types of use within mathematics teaching. In some cases, trainees use accurate mathematical vocabulary, other times they use words which have one meaning in mathematics but a different use in everyday use, and some words were used in what might be called an 'eccentric' way. Further specific research into use of these types in the classroom could be enlightening.

## 5.7 Contribution to the field

This study started from the point of reflecting on mathematics teaching in schools and then in higher education for teacher training programmes. It developed following involvement and engagement in the field of primary mathematics education, largely through attendance and presentations at conferences, seminars and research colloquia. The motivation behind this study lay in a desire to engage at a critical and analytical level with the process of mathematics planning, teaching and learning after being involved in the process as a trainee teacher, then as a teacher, and most recently as a tutor and lecturer. The research presented here comes from bringing together two key pieces of existing research, those of the Knowledge Quartet (Rowland *et al.*, 2009) and the work of Watson and Mason (2005) examining examples within mathematics teaching. By analysing the planning and choice of examples of a group of trainees from one institution, some key issues about how trainees perceive the theoretical side of their course and relate this to their classroom practice have been uncovered.

The conclusions described earlier in this chapter have highlighted the various approaches that one group of trainees use in their practice as teachers of primary mathematics, and given some of the outcomes of the interviews and lesson plan analysis, it might be interpreted that there is scope to extend the findings across a wider sample as a means of enabling more trainees and their course tutors to consider how the issues of planning mathematics and choosing examples is best addressed for future cohorts.

In carrying out the study in one designated university with trainee primary teachers from one course, analysing lesson plans from their final school placement and interviewing a sample group before selecting seven case studies of different ability trainees, it was possible to obtain a unique and original perspective on the research questions through the trainees involved in the study.

## 5.8 Final Remarks

At the end of the study it was possible to combine the outcomes from the three research questions to get an overall picture of how the case study trainees behave when planning mathematics lessons, how they draw on knowledge, understanding, theory and practice to produce lessons with teaching examples and exercises for children to use for developing their conceptual understanding of mathematics in primary school.

On the evidence presented here, drawn from the data collected in lesson plans and interviews, it can be suggested that trainee primary teachers generally plan their mathematics lessons using generic planning guidance instilled by their institutional course. This often includes using the current non-statutory government guidance, the Primary National Strategy, through its Framework for Mathematics and also drawing on mathematics scheme textbooks and online resources, for example, the Abacus Evolve scheme. There seems to be an emphasis on ensuring that the plans contain information under a range of headings to meet university requirements, even if much of the information does not translate directly into effective teaching of mathematics.

Lessons focus on 'meeting objectives', a phrase that permeates the target-driven culture of education in 2010, and the pedagogy of teaching mathematics tends therefore, to become hidden behind the bureaucracy of ITT and its obligation to meet the standards set by the Office for Standards in Education (Ofsted) and, where relevant, the Quality Assurance Agency (QAA). In terms of choosing examples for mathematics lessons, the evidence seems to suggest that trainees rely on examples from internet or text sources, modifying them only in what they see as appropriate ways for their knowledge of the children's abilities. When they select their own examples to offer children explanations or practice exercises, the examples are taken from existing ones on the assumption that they must be good examples, and when trainees choose to make their own examples, they tend to be randomly selected with some reference to the trainees' subject knowledge, but often lacking deep understanding or pedagogical direction. It is hoped that the outcomes of this study, however limited, might prove to be effective in determining further research in future, with the aim of improving trainees' choices of examples.

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**Appendix 1**

**Information Sheet: Research into Choice of Mathematical Examples 2007-08**

Dear Student,

I am contacting you to see whether you would be willing to be a participant in a piece of research which will help me complete my EdD (Education Doctorate).

The objective of this research is to try to get some insight into how Y3 trainees on their final School Experience chose mathematical examples for teaching, and to devise a framework that will assist future trainees in selecting appropriate examples.

Who Am I?

I am Ray Huntley, former primary teacher/headteacher and maths lecturer and currently head of CAWNS at the University of Gloucestershire, as well as being an Education Doctorate student, but no longer a member of staff in the Education courses. As an education academic I have more interest in qualitative research than quantitative research and this academic year I have begun to research how trainees choose the mathematical examples in their lessons.

What will the research entail?

First of all I intend to collect as many examples of mathematics lesson plans as possible from Year 3 trainees, from during the final school experience undertaken between January and March 2008. Following some initial data analysis of the lesson plans, I will select a sample of about 15 students based on data that includes maths interview test scores, maths assignment grades in Years 1 and 2 and GCSE maths grades. The sample will include male and female students from Key Stage 2 placements in 2008.

Once the sample is selected and participant consent has been received, I am planning to do short interviews which will take from 30-40 minutes, in a location of your choice and will be recorded. During the interviews I hope to cover topics such as your reasons for choosing your examples in maths lessons during your final School Experience. I will also want to discuss with you the way you used the examples and how effective you thought they were from a pedagogical perspective, as well as how you might choose examples for lessons in future. In return, you may benefit from the feedback I would give as a form of individual personal development to support your future mathematics teaching.

Confidentiality

As the researcher for this study I will be the only person doing the interviews with you. I will need to have record of your consent to do the research, both your consent to lend me your lessons plans, and also to be willing to take part in the interviews if selected, but the signed consent form will be kept in a locked filing cabinet that only I have access to.

Following the research interview it will be transcribed, during the transcription process your name will be changed to a pseudonym, and in the reporting of the research only your pseudonym will be used. You will be entitled to read the transcript for accuracy. On the consent form I will also ask you to keep confidentiality about the interview content.

Analysis

I will conduct an analysis of the transcript material, to see if there are any emergent patterns with regard to the issues that I am interested in, and write this up as part of my doctoral thesis.

**Appendix 2**

**Students Consent Form 2007-08**

I understand that my participation in this project will involve me submitting for analysis mathematics lesson plans from my final school experience during January to March 2008, and following some initial data analysis by Ray Huntley, possibly taking part in a short (approximately 30-40 minutes) interview about my choice of examples in the some of the mathematics lessons. During the interview the researcher will ask about the lessons I planned and the examples I selected for teaching, as well as a reflection of how successful I thought the lesson was.

I understand that my participation in this study is entirely voluntary and that I can withdraw from the study at any time for any reason without any penalty.

I understand that I am free to ask questions at any time. If for any reason I experience discomfort in any way I am free to withdraw or discuss my concerns with Ray Huntley (rhuntley@xxxx.xx.xx).

I understand that the information provided by me will be held anonymously such that it is impossible to trace the information back to me individually. The recording and consent forms will be kept securely by Ray Huntley in such a way that no one else will have access to them. The recording will be erased as soon as practical, and before the final report of the research is written. The interview transcript will only refer to me by a pseudonym and while the interview transcript will be retained no link between it and my actual name will be kept.

I, *(student)*

consent to participate in the study conducted by Ray Huntley, *University of Gloucestershire* by providing mathematics lesson plans.

Signed (Participant): Date:

I also give consent to being interviewed if selected.

Signed (Participant): Date:

Signed (Researcher) Date:

## Appendix 3

**EdD Interview schedule – Year 3 B.Ed Student sample – Summer 2008**

Introduction

Thank you for agreeing to be interviewed today. The purpose of the interview is for me to ask you about some of the lesson plans and supporting materials that you have already provided, and to ask you about the processes you use when planning for teaching and learning in mathematics. Everything discussed will be recorded and kept confidentially, and you will be provided with a copy of the transcript.

Are you happy to continue?
(Start recording)

*This interview is with research participant number EDXX on (date)*

Warm-up

Did you enjoy your final school placement?

What year group did you teach? What were they like?

Do you have a particular good memory from the placement?

Main body

*I'd like to ask you about planning for mathematics lessons.*

1. How do you go about planning for a mathematics lesson?

2. Which resources have you used to help you plan mathematics lessons? In what ways were they helpful?

3. Have you used the interactive planning tool on the PNS online mathematics framework? Was it helpful and how?

4. During your placement, how did the school mathematics scheme influence your planning?

5. To what extent do you think the choice of examples in mathematics lessons is important for children's learning?

6. If you use a published scheme, either for teaching ideas or activity sheets, how much consideration do you give to the examples offered and the sequence they are presented in?

7. Have you ever changed the given examples for ones which you feel are better? Can you recall any particular examples of this?

8. When you decide on examples to use in your teaching input, what factors do you consider?

9. If you design your own worksheet for groups of pupils to use, how do you decide the sequence of examples on the sheet?

10. Which theories, models or literature are you aware of that offers guidance about choosing examples?

11. If you knew there was some theory-based guidance for choosing examples, would you look for it and consider using it?

*I'd like to ask you know about your own mathematical subject knowledge.*

12. Would you say you feel confident about your mathematical subject knowledge?

13. To what extent do you think your level of subject knowledge helps you choose appropriate examples?

14. How do you try to improve your level of mathematical subject knowledge ahead of teaching a particular topic?

15. How do you think mathematical subject knowledge can help in choosing appropriate examples?

<u>Cool-off</u>
*Just some final questions about teaching and learning mathematics.*

16. What have you learned from planning mathematics alongside experienced teachers in your placement schools?

17. How has this helped you in planning your own lessons and choosing examples?

18. What have you learned from the mathematics modules on the B.Ed course about how to choose appropriate examples in mathematics lessons? *(and finally…)*

19. Do you think you would have benefited from more guidance on choosing examples as part of the course?

<u>Close</u>
That's the end of my questions – is there anything you would like to add about how you choose examples for mathematics lessons? Would you like to ask me anything?

Thank you for taking part – the interview is now ended.

(Switch off recording)

## Appendix 4

### Information Sheet: Research into Choice of Mathematical Examples 2008-09

Dear Student,

I am contacting you to see whether you would be willing to be a participant in research which will help me complete my EdD (Education Doctorate).

The objective of this research is to try to get some insight into how Y3 trainees on their final School Experience chose mathematical examples for teaching, and to devise a framework that will assist future trainees in selecting appropriate examples.

Who Am I?

I am Ray Huntley, former primary teacher/headteacher and maths lecturer and currently head of CAWNS at the University of Gloucestershire. As an education academic I have more interest in qualitative research than quantitative research and last academic year I have begun to research how trainees choose the mathematical examples in their lessons.

What will the research entail?

I will select a sample of students based on the year group and maths topics to be taught during final school placement. I intend to collect examples of mathematics lesson plans from that placement, and examine data that includes maths interview test scores, maths assignment grades in Years 1 and 2 and GCSE maths grades. The sample will include male and female students from Key Stage 2 placements in 2009, and may include the opportunity for some classroom observation, subject to clarification with the university authorities and the school.

Once the sample is selected and participant consent has been received, I am planning to do short interviews which will take from 30-40 minutes, in a location of your choice and will be recorded. During the interviews I aim to cover topics such as your reasons for choosing your examples in maths lessons during your final School Experience. I will also want to discuss with you the way you used the examples and how effective you thought they were from a pedagogical perspective, as well as how you might choose examples for lessons in future. In return, you will benefit from the feedback I would give as a form of individual personal development to support your future mathematics teaching.

Confidentiality

As the researcher for this study I will be the only person doing the interviews with you. I will need to have record of your consent to do the research, both your consent to lend me your lessons plans, and also to be willing to take part in the interviews if selected, but the signed consent form will be kept in a locked filing cabinet that only I have access to.

Following the research interview it will be transcribed, during the transcription process your name will be changed to a pseudonym, and in the reporting of the research only your pseudonym will be used. You will be entitled to read the transcript for accuracy. On the consent form I will also ask you to keep confidentiality about the interview content.

Analysis

I will conduct an analysis of the transcript material, to see if there are any emergent patterns with regard to the issues that I am interested in, and write this up as part of my doctoral thesis.

## Appendix 5

### Students Consent Form 2008-09

I understand that my participation in this project will involve me submitting for analysis mathematics lesson plans from my final school experience during January to March 2009, and following some initial data analysis by Ray Huntley, possibly taking part in a short (approximately 30-40 minutes) interview about my choice of examples in some of the mathematics lessons. During the interview Ray will ask about the lessons I planned and the examples I selected for teaching, as well as a reflection of how successful I thought the lesson was.

I understand that my participation in this study is entirely voluntary and that I can withdraw from the study at any time for any reason without any penalty. Taking part in this study will not affect my degree grades, either for assignments or for school placement, in any way.

I understand that I am free to ask questions at any time. If for any reason I experience discomfort in any way I am free to withdraw or discuss my concerns with Ray Huntley (rhuntley@xxxx.xx.xx).

I understand that the information provided by me will be held anonymously such that it is impossible to trace the information back to me individually. The recording and consent forms will be kept securely by Ray Huntley in such a way that no one else will have access to them. The recording will be erased as soon as practical, and before the final report of the research is written. The interview transcript will only refer to me by a pseudonym and while the interview transcript will be retained no link between it and my actual name will be kept.

I, *(student)*

consent to participate in the study conducted by Ray Huntley, *University of Gloucestershire* by providing mathematics lesson plans.

Signed (Participant): Date:

I also give consent to being interviewed if selected.

Signed (Participant): Date:

Signed (Researcher) Date:

**Appendix 6**

**<u>EdD Interview schedule – Year 3 B.Ed Student sample – Summer 2009</u>**

<u>Introduction</u>
Thank you for agreeing to be interviewed today. The purpose of the interview is for me to ask you about some of the lesson plans and supporting materials that you have already provided, and to ask you about the processes you use when planning for teaching and learning in mathematics, in accordance with my research questions.

1. What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?
2. How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?
3. Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?

Everything discussed will be recorded and kept confidentially, and you will be provided with a copy of the transcript. Are you happy to continue?
(Start recording)
*This interview is with research participant number ED on (date)*

<u>Warm-up</u>
Did you enjoy your final school placement?
What year group did you teach? What were they like?
Do you have a particular good memory from the placement?

<u>Main body</u>
*I'd like to ask you about planning for mathematics lessons.*

What pedagogic considerations do a cohort of trainee primary teachers use when choosing mathematical examples in the classroom?

1. How do you go about planning for a mathematics lesson?

2. Which resources have you used to help you plan mathematics lessons? In what ways were they helpful?

3. Have you used the interactive planning tool on the PNS online mathematics framework? Was it helpful and how?

4. During your placement, how did the school mathematics scheme influence your planning?

5. To what extent do you think the choice of examples in mathematics lessons is important for children's learning?
6. If you use a published scheme, either for teaching ideas or activity sheets, how much consideration do you give to the examples offered and the sequence they are presented in?

7. Have you ever changed the given examples for ones which you feel are better? Can you recall any particular examples of this?

8. When you decide on examples to use in your teaching input, what factors do you consider?

9. If you design your own worksheet for groups of pupils to use, how do you decide the sequence of examples on the sheet?

How do these pedagogic considerations fit within current theoretical frameworks in primary mathematics pedagogy?

10. Which theories, models or research literature are you aware of that offers guidance about choosing examples? ELPS? VAK? Knowledge Quartet?

11. Relate items on plans to K4tet…

12. If you knew there was theoretical guidance for choosing examples based on research, would you consider using it?

*I'd like to ask you know about your own mathematical subject knowledge.*

Is there a relationship between the cohort of primary trainees' level of mathematical subject knowledge and the types of examples they select?

13. Would you say you feel confident about your mathematical subject knowledge? Why? How could you improve your confidence in mathematical subject knowledge?

14. How would you define mathematical subject knowledge? What does it include?

15. Do you think your level of subject knowledge helps you choose appropriate examples? How?

16. How do you try to improve your level of mathematical subject knowledge ahead of teaching a topic that you are not confident in?

Cool-off

*Just some final questions about teaching and learning mathematics.*

17. What have you learned from planning mathematics alongside experienced teachers in your placement schools? Can you give examples?

18. Has this helped you in planning your own lessons and choosing examples? How?

19. What have you learned from the mathematics modules on the B.Ed course about how to choose appropriate examples in mathematics lessons? *(and finally...)*

20. Do you think you would have benefited from more guidance on choosing examples as part of the course? How?

<u>Close</u>
That's the end of my questions – is there anything you would like to add about how you choose examples for mathematics lessons? Would you like to ask me anything?

Thank you for taking part – the interview is now ended.

(Switch off recording)

## Appendix 7

## Letter re: access to Diagnostic Numeracy Test results 2008-09

Dear Student,

Re: Doctoral Research into Choice of Mathematical Examples

I have contacted you previously to see whether you would be willing to be a participant in a piece of research which will help me complete my EdD (Education Doctorate) into how Y3 trainees on their final School Experience chose mathematical examples for teaching, and to devise a framework that will assist future trainees in selecting appropriate examples.

I have already collected over 400 mathematics lesson plans from 22 Year 3 trainees, and following some initial analysis of the lesson plans, I interviewed a sample of 10 students based on data that included maths interview test scores, maths assignment grades in Years 1 and 2 and GCSE maths grades.

In order to further my analysis, I need to access data held by the University from the Diagnostic Numeracy Test which you took when you joined the course. The purpose of this is to compare the results of the sample that provided lesson plans and interviews with the whole cohort, to determine whether the sample is representative of the cohort as a whole.

Confidentiality

**I need to separate the sample group results from the whole cohort results, and this cannot be done from a full anonymised set of results. I therefore need to know which result came from which student, and for this to happen I need to ask your permission to access your test result.**

Once I have access to the data, no individual's result will be identified in the research or subsequent work that follows from it and so there are no implications for you as a result of me accessing your Test score.

**Appendix 8a**

**Interview transcript – ED06 'Suzy'**

R: This is interview with research number 006. First of all did you enjoy your school placement?

S: Yep, it was difficult but I enjoyed it.

R: Difficult?

S: Yes, a bit of a challenging class.

R: And what year were they?

S: Year four and five.

R: And what were they like? Tell me a bit about them.

S: Well, they were quite boisterous, they came from difficult backgrounds, so that was probably why, and they were all pretty different, well the year fours were higher than some of the year fives, so it was completely mixed up.

R: Have you got a particularly good memory from the placement? Something positive that you remember.

S: Yes, on the last I took them all up Cleeve Hill, walked the whole way, because I wanted them to have something that they felt they had achieved, 'cause most of them never get to go up there, so that was fun.

R: Good. Let's talk about maths now. Suppose you had to teach a maths lessons next week, how would you go about planning a maths lesson?

S: Is it after being given the topic?

R: Yes, choose a topic, say you were doing year five multiplication.

S: Erm, yeah, well probably...I'd have to check what they already knew, by finding out from the class teacher, and then I'd go into the maths and have to practise it to make sure I know what to do, 'cause they all change and stuff...so I'd sort that out, and then...erm...I don't know, sometimes I go for the starter first, just to try and imagine the lesson what I want., or what I'll be doing, and whether you are going to need more time for the teaching point or...yeah, and then check the text books they are supposed to be using, otherwise if it's not, then I'd make...

R: So, you said...you mentioned textbooks, on placement, did the school have a scheme that you used?

S: They were trying to use the national…the new strategy, and my teacher was head of maths, so he was…yeah, he tried to help me use the planning thing…

R: Oh is this the online planning tool?

S: Yeah, but I didn't really manage…so he told me basically which…the…some of the sections in the book that we could use, but then towards the end of the placement I decided to try and find it, and so a lot of the time I just did read through and write up a lot of my own questions for them.

R: Just thinking then about the examples that you use, so if you're going to do some direct teaching to the whole class, you must have some idea of which questions you're going to use to demonstrate to the children. How do you decide what examples to use to demonstrate things?

S: You mean about like, which questions?

R: Yes, which questions are you going to use?

S: Sometimes I started with an easy method first, and then go through it on the board, so I model it, then I got them to write it down in their book because a lot of them struggled with the concept, they were also…if everyone was doing it, and then if you were recapping I would get one of the children to come up and help do it.

R: Do you think it's important which examples you use, which questions you use, or can you just have a random choice?

S: Well, I think definitely to make sure it works.

R: Yes!

S: I did one of the 'Countdown' ones and found out that it was impossible! But yeah, I think it is, and also to make sure there is enough, 'cause with the differentiation, you get one that everybody could actually work through.

R: If you used a textbook, say, to get examples off, either for teaching or for worksheets, how much do you look at the questions in the book and decide how good they are for what you need? Do you sometimes take them as they are, or do you sometimes adapt them and change them?

S: When I'm planning?

R: Yes.

S: Yeah, I usually check them through and decide if…'cause I was trying to keep in mind all the different children as well, and who would manage what, and then I'd choose those, and maybe a certain group could just do from the book, but just…I'd try to pick out certain questions for them to do rather than just work through it, and then otherwise, write up some…they did end up on a worksheet format, but at least it was more individual.

R: And when you write them down, do you just make them up randomly or do you have a reason why?

S: I try to work them out so that if it was trying to include different, like maybe different numbers, like some two digit and some harder.

R: If you see some questions in a book, do you sometimes change them for ones that you think are better?

S: Sometimes I photocopy them, and then for some of the children I give them certain bits of the questions.

R: And if you decide to use some questions for teaching the whole class, what sort of things go through your mind to help you decide what questions to teach? Do you make them up out the top of your head at random?

S: Sometimes…usually I have one that I've written on my plan to start with, and I'd see how they go, and then see if we need to…and often then I would just make them harder.

R: And if you design a worksheet for different groups or for a whole class, how do you decide the sequence of the questions? Does it matter?

S: Yeah…sometimes I try to do…I definitely, if I was doing, like, multiplication, I wouldn't do all the two times, I'd try to mix it sometimes, and then other times I'd a couple of…for the low ability, I'd do a couple of the same, so you get used to it, and then the next ones…

R: So it's generally done on ability?

S: Yes.

R: Have you come across any theories, or models by anybody, or literature you've read, that gives guidance about choosing good questions?

S: Not that I can think of…(laughs)

R: If there was such guidance in a book, if there was, say, 'Huntley's model for choosing good questions', is it something you think you'd make use of?

S: I think so…because I know, like, questioning in general, is something I'd like to improve anyway, but yeah, no, choosing the questions, 'cause otherwise you end up doing it from just what you think, rather than having guidance…

R: Have you seen any Numeracy Strategy material about choosing good questions?

S: I'd like to say so, but it would be lying!

R: A couple of questions about maths knowledge then. How confident are you about your level of maths knowledge for teaching?

S: Erm...actually, I should be better than I am because I did 'AS' level, but then it's still different. I think the thing that has thrown me back is how the methods of laying it out has changed, because the way I learnt it, and now I'm having to re-learn it, so that would probably be the same, but...

R: But you feel you're okay with what you...

S: Yeah, kind of.

R: So if then, you need to teach a topic where you are not so confident, take an obvious one, say fractions, most people have trouble with fractions, what would you do to go about refreshing your knowledge on fractions before you taught it?

S: I'd look up my own revision notes that I kept at home, from GCSEs and stuff, I could go through that, and also, erm...well, I'd look in other books as well, to see what...how they describe it, but some of them, I found I didn't actually understand that so then I'd have to ask staff, like you...

R: Would you use books like Haylock or similar ones?

S: Yeah, I have used that one actually.

R: Do you think having a good level of maths knowledge would help you choose better questions?

S: Yeah.

R: You think there's a link?

S: Yeah, I think so. I think if you understand the thinking behind each question, then you'd know, yeah, which ones are better.

R: In placements, you've obviously been alongside experienced teachers and seen them planning and teaching to some extent. What sort of things have you learned from experienced teachers about teaching maths?

S: Erm...
R: Can you think of any teacher where you thought 'Cor, that's a good idea, I'll remember that' or 'I like the way they've planned that.'

S: Yeah, with like, one class, I ended up being with an NQT, but he was trying out loads of new ideas which was really good, he was doing the whole measuring thing, and he brought in a whole (not clear) but he made it into like, a discovery session as well, and I just thought having seen that, compared to having it all written and copying it down, yeah that's definitely good.

R: Finally, thinking about the maths modules here, are there things that you can think of from those maths modules that have helped you be able to choose good questions in your teaching?

S: For the starters, for me have been one of the biggest help, 'cause actually it's hard to start up questioning in general. I'm not sure in terms of other help…I don't know, in terms of like the whole investigation side, then yeah.

R: And finally do you think if there was a specific session on how to choose good questions for worksheets, or how to choose good questions when you are directly teaching, do you think you would have benefited from that sort of thing?

S: Yes I think I would, because it's something you don't get to actually think about so much…sometimes you do it, but you don't always think about it, so you don't put so much effort in.

R: That was the last question, is there anything else, as we've been talking, that's perhaps come up in your head about maths teaching, anything you want to add? Any experience you've had about teaching maths, or a good worksheet you remember?

S: One thing that just came up for me was, 'cause I'd done quite a series of sessions, and then at the end I decided just to do a kind of a test, but not under the name of a test, but the children who had done really well, and still at the time couldn't do anything, they'd forgotten everything that I'd actually sat down to do, so that kind of just highlighted for me that actual aspect about how sometimes you can think a child knows it, and they do, but afterwards you find out actually they can only do…

R: How do you think you can get over that sort of thing?

S: Maybe doing more going back to the topics, rather than just doing it the once and then moving on without them going back afterwards.

R: And do you think better questions might help children learn better and retain it?

S: Yes, 'cause if they are questioned more about it as well, then they would have to know it better.

R: Thank you.

**Appendix 8b**

**<u>Interview transcript – ED18 'Sharon'</u>**

R: This is interview with research participant number ED18, so did you enjoy your school placement?

S: Yes, it was really, really good.

R: What year group was it?

S: Year 4.

R: And what were they like?

S: Erm…the class I taught were of quite a good ability actually, it was a large school, so there were 4 classes, so the class that I had were actually quite good ability across literacy, and then for maths they were streamed so I had the lowest set, and there were quite a lot.

R: Okay, have you got a particularly good memory from that placement? Perhaps a maths memory that stands out?

S: Yeah, I really enjoyed teaching them because…just the fact that they were low ability, I'd never worked with low ability, I'd always had high ability, so I was a bit anxious about teaching them because I didn't know if I'd pitch it too high all the time, and obviously you have to be even clearer when you explain everything and they did understand it, and we did a test, to sort of find out how much they'd learnt while I taught them, we did it on one of those…electronic ones…they are like voting pads, we did it that way, and they all got, apart from one, at least 17 or above out of 20, which was quite a good feeling, 'cause you know that they have actually processed it, whether they'd still remember it in a few weeks I don't know but some if it had been a couple of weeks ago, before…so that was really good.

R: Excellent! Let's talk about planning maths lessons. How do you go about planning for a maths lesson?

S: I…on this placement I spoke to the teacher about what topics we were going to cover, because they'd started using the PNS, but they didn't have any, sort of, strategies in pace as such, and so we literally had the Year 4 PNS out and thought, right, we need to cover everything on this, what should we do, and with him we worked out which topics I was going to do then I went away and planned them, then I sort of looked on the internet at Primary Resources and things to see if there were any fun things and…internet…I like to use interactive whiteboard games, even with them…so it was really good 'cause there was like, subtraction bingo and things like that which just sort of like get them going. That's the way I plan.

R: It's interesting the way you said 'even with them' as though they are perhaps not for older children?

S: Yeah, I'd say because Year 4...because normally I would use them with Year 2,but even with Year 4 they still really enjoy it and I just use ones that I pitch higher and they don't see it as silly,even though they are in Year 4, so it was really good fun.

R: Have you used the interactive planning tool on the PNS?

S: I've had a brief look at it...there was a staff meeting and they showed us...they'd been on a maths planning meeting and they showed us...there's a big CD with loads and loads of planning on it and they went through it and showed us the grid that you can, sort of like, stick like maps and things on it, but I hadn't actually used it through that, I used it through the paper copy because I don't particularly like having it like that...

R: So was it helpful at all, looking at the online framework?

S: It was...because it was towards the end, I'd already done most of my planning, but there were some ideas on there that I'd definitely use and I did actually make a grid of like 9 squares with the questions on, and then put like 9 numbers underneath and I had that on a display towards the end, and I think it's a good way of having an interactive display that if they finish, you just send them off, it doesn't matter what topic you are doing, you can just go off and have a go, and it sort of keeps them quiet without disturbing you, and you can help the ones that are struggling.

R: During the placement, did the school have a maths scheme that influenced your planning?

S: No, they were...they've recently been doing a lot on their literacy and that was really, really good, and they'd just taken on the PNS and the new maths co-ordinator was trying to sort it all out and they were talking abut looking for a scheme of work and things, because they said it's really hard to know what to teach when and because it's just a list of...whereas with the NNS it sort of like, breaks it down more for like terms, it's just the objectives and they didn't know when they should teach different topics, except that then it was really good because you can choose what you want to teach and when, you don't have to...'cause we started off teaching them the basic, like, addition...and he taught them and then I did subtraction and multiplication, and sort of like building up their basic concepts before doing all the other stuff, whereas it may well have been that would have come at the end if it was tailoring it to the class a bit more.

R: Did they have an older scheme that they still dipped into, or have they sort of just gone to PNS?

S: No, I think they'd just gone to PNS because it didn't link over very well. I don't think, 'cause they did have a scheme before and that's why they were keen to get another one and because it would be really helpful for the teachers in knowing what to teach and when.

R: Thinking about the examples you use in maths lessons now, to what extent do you think the choice of examples that you use is important for children's learning?

S: I think…yeah, it's really, really important, particularly thinking of subtraction…I was doing subtraction…and we were doing it…and they had to subtract a number below from the higher number, so I deliberately chose, like, 99 and all the digits were lower,'cause the teacher was in the lesson and said to me, he was like, 'Are you not worried that they are going to start having a bigger number?' and now I've covered that, so I'd already covered the fact that they would then be able to just pick out a number at random, 'cause it's a bit more fun,and I knew that they wouldn't have a number that was higher than a 9, so I did…so I think with that, if I'd have not thought of that, they would have come across a lot of problems, because they hadn't actually come on to the fact of subtracting of 10 and I wasn't going to cover that with them because they were struggling anyway so we were going to leave that, but you do have to think about it, and if you don't challenge them enough then they can't do it, like with multiplication, I only did…they had numbers, but they were only multiplying by 2 or 4 because they didn't have basic knowledge of the subtraction tables, so it was pointless doing it times 8 because they couldn't do 2 times 8, so they couldn't actually do the longer multiplication anyway.

R: If you used scheme ideas or PNS ideas for either for your teaching, or for children to do sheets and things, how much consideration do you give to the examples that they offer?

S: I sometimes alter them…like, I got one for homework from one of the Primary Resources site because I wanted them to do time, it was to do with, like, how many seconds are in an hour, just to sort of recap that knowledge without having to go over it in class, and then at the bottom I put on my own, and sort of like adapted it and wrote it out myself so that it would meet the needs that I wanted, and I found a game and I put some questions with it as well, for another topic.

R: So what was it when you saw it that made you think, 'I'm going to change this'?

S: The questions…if they are pitched too high, I just thought, they are not going to understand it, 'cause it was aimed at Year 4, but as low ability, they are not necessarily working at Year 4 level, so it was just too hard, and it can be as simple as changing the numbers, it doesn't have to be that difficult, and I wrote out my own sometimes, just so that I could get to set out the working how I wanted them to do it, through setting out the lines, and I think it's better to make your own, but not to just do it for the sake of it, if you're just going to give them a worksheet, there's no point in spending hours agonizing over it…find it on the internet and adapt it so that it will meet their needs.

R: Can you think of any examples where you have done that? Or the sort of things you might have done?

S: More looking at…a couple of sheets, and sticking them together, if that makes sense, so I looked at a couple of times sheets and then I put them together and…I looked at a worksheet for a board game which I decided to do with the teacher, 'cause he suggested it, then I found a blank game outline and then I wrote my own questions to go with it and what I wanted them to put on it…so it was to do with subtraction, so I used their initial worksheet, but with my own instructions so that its tailored to them, so it didn't take me ages to prepare it, but it was a really good thing for them to do, and then it went up on the display and they really enjoyed it, and it was getting them to practise using the words that are used for subtraction, so 'take away 3', and they actually had to write them on, so it's their processing it as well.

R: Just thinking now about when you do your direct teaching, either whole class or to a group, and you choose some examples to demonstrate a point with, what sort of things…what sort of factors do you think of when you are choosing those examples?

S: I normally start off quite easy because it's better to start off with a really easy one that they can do, because they already get the confidence up that they can do it…if they can't do what you think is easy, 'cause you can sometimes pitch it too high, if you think that's easy they can still struggle with it, so then you can lower it or higher it as you need to, and obviously, like, language and stuff, I know that I was one of those people that used 'sum' instead of calculation and my teacher sort of said 'oh, someone's mentioned this to me and you're doing it as well', and every time I used it, I was really conscious of it after that and thought 'No…can't use that word!', so, just…sort of like, the language and things, especially for doing my dissertation, are quite important.

R: So the language and the ability level?

S: Yeah.

R: And then if you design a worksheet, how do you decide the sequence of the examples on the worksheet?

S: Normally…with those children, I tended not to go off too much, and tended to do the same thing for the lesson because it often took quite a while to explain it, in my own planning I would always write out, so instead of using a worksheet I wrote the questions on the board sometimes, so then I could go to the children who'd finished and write questions in their book, and I would get more…progressively more difficult, so they were all doing 2 times tables for multiplying and then I'd give some 4 times tables, and then we were doing angle work and I had to organise the angles, and then I had in my head, I knew what I was going to do, I was going to get them to try and draw them so they drew easy ones and then, like, I went over and wrote on their books, like, draw a 180 degree angle, and they were able to, sort of, adapt it to that so that you weren't ...I knew in my planning rather than actually telling them, like if you've finished this you will be doing that, so that those who didn't get on to it didn't know what the others were doing anyway so they didn't know they were behind as well, which is quite, you know…

R: So when you go to their book and write an extra example in, is it a sort of random example or is there a reason why that example follows what they've done?

S: Not…I don't necessarily plan exactly what numbers I'm going to use, but, like, for example, if it's multiplying and I knew I was going to multiply it by 4 then I deliberately wouldn't pick difficult numbers because it just gets too hard for them, but with the angles I knew which ones I wanted them to do and knew I wanted them to start with 180 and 90 and if they could do that, give them a go at doing a 45 degree angle because that's more difficult than the other two.

R: Are there any theories or models or literature that you know about where it gives you guidance on how to choose good examples? Have you come across any reading at all that says 'this is a good way of choosing an example', or 'these are good examples and these are not.'

S: There's…there's quite a few good books, 'cause I did my dissertation on maths, so there's quite a few good books…but a lot of them, I know, is it Cockburn, she has quite a good book for examples of subtraction, and she's got a big mind map to say these are the potential misconceptions that they might have, because if you don't think of them, then they are going to come up, and I'd already thought of most of them before I looked at it, but it gives you confidence that you know you've sort of dealt with them. I know they were there for time as well, and Chinn's got a few good ideas as well…

R: And if you knew there were other theory-based guidance documents on choosing examples, is it something you think you'd have time to consider and look at and adapt?

S: I think now while I'm at uni, yes, but not necessarily when in school because no one introduces you to these people, like I only knew about Liebeck because we were taught it through maths, and through the idea of actually following it through and it's something you'd probably do but without knowing that it's related to a person's theory, and a lot of teachers don't know about these people because no one comes in and no one tells you about them and tells you who to read about.

R: That's interesting because what I'm looking at is how examples affect people who are on a course like this, and I'm wondering about how it's different to what practising teachers think. Let's talk about subject knowledge…how do you feel about your level of subject knowledge…do you feel you are reasonably confident in most areas for teaching?

S: Yeah, I think I can do the basic maths, I sometimes have to read up on it purely because I've done A-level so also I sometimes skip out the easy bits and I can't remember quite how to teach it to children because a first you think 'Why don't they understand? It's really easy'…that was something that was good with having low attainers because it made me go back to real basics, and then you can adapt it from there whereas with highers you don't have to worry so much about your explanations because they can get it that little bit easier.

R: Do you think your subject knowledge helps you choose good examples, compared to somebody who, say, hadn't done A-level and wasn't so confident?

S: I don't know, I think I have to be more aware of what I choose because I would probably pitch it too high every time, and I know that that was something I kept thinking of and being really aware of, don't pitch it too high because you are used to working with higher attainers as well, and I think it helped me think about my examples a lot more whereas someone who struggles with maths a little bit may...really thoroughly plan a maths lesson and really think about those things whereas I could kind of carry it off, 'cause I've done Year 6 and they were doing SATs papers, and I went in...for me it was okay because I had the basic knowledge there, and I could help them work through the problems because it was Year 6 level anyway, and it was okay for me, but someone who did struggle with maths, they would probably want to read through it and recap it...so it sort of works to and against you, I think.

R: Have there been any topics that you've had to teach, say those Year 4s, where you've had to brush up on your knowledge for a particular topic?

S: For subtraction I looked over the different ways of teaching it, so there was like, the extended column method and there was the column method and then obviously doing the mental work, and using number grids to subtract tens and units and sort of, rather than just teaching 'subtraction', teaching it a range of ways so that they can choose which way they find the easiest, and I found that...initially I thought that the extended column method would actually be easier for them, but it wasn't, because they kept...they kept not remembering the zero, that it was in the tens column and they'd just write 7 minus...and they'd end up crossing it out, whereas if it was just the column method, they were okay, they could cross off the 1 with their hand and then just take it away from below and it was easier because it was just numbers and they didn't have to think, like, 'Oh, it's actually 30', but when they looked at the answer they knew it was 30 but they didn't have to process that as well.

R: When you say that you looked at some...you looked up how to teach different ways of subtraction, where did you go to look for those ways? What did you use?

S: I spoke to the teacher because he'd taught Year 4 quite a lot of times, so he knew, and then I looked on the internet and then in Haylock's book, just to sort of like see if there was any ideas in there, and then once I knew the basic, once I knew what it was, I didn't have to read how to teach it, 'cause it just clicked, and I only had to look at, say, one example which does help, but having to sort of just quickly find out what he meant by the extended column method, 'cause I couldn't remember!

R: Do you think that having better subject knowledge should generally help people choose better examples?

S: I don't know…I think it's one of those mixed topics, I think it should, because people would say, 'Well if you're better at maths then you should be better at teaching maths', but it's different understanding it and teaching it, to understand it you just have to grasp concepts, but to teach it you have to be able to explain how you got it which is a completely different process, and I know that it's something I found harder at first, especially going back to the real basics, because you do forget about them, 'cause you start doing silly things like integration and then you can't go back to just doing the simple things so now after like three years of going back to it, it's okay because I know what to do now, and I wouldn't probably be able to go back to integration again, but I think that for people who do struggle with maths, they probably are more prepared for a maths lesson because they want to read up on it, whereas I would probably feel that's an area of strength so I wouldn't necessarily take as much time on planning that subject, I'd do something that was more a weakness for me that I'd have to read up on.

R: Just some final questions about learning and teaching maths generally…from your placements, what have you learned from watching experienced teachers plan maths?

S: They tend to use schemes of work, from what I found, but it's something that I've tended to take on because it's one of my interests, so I've kind of taken on most of the planning, so I haven't necessarily seen it being planned, because I've said I want to teach that as part of my percentage, so you know, I haven't…I found a lot of worksheets and one thing that I didn't like was in Year 2, was that they had a big book and they just photocopied worksheets and there was just…the differentiation was just how much help and what resources they had to help them with it, and I didn't think that worked…in my own personal experience I don't think that's a good way of doing it…and I decided I was going to plan much more exciting activities, and sort of, go against the grain, but obviously they have to cover the same as another class, so you can't always do that, it depends on the school.

R: So from your approach to planning, because it's something that you're interested in, what have you learned from the way you did it that's helped you improve your maths teaching?

S: Probably…just examples of ways of teaching, because I tend to do a big spider diagram, say if I'm doing time, a big spider diagram of all the things I might cover, and then look into how I could cover them and I would choose, say, the few more interesting ones and leave the other ones to…sort of like, last…because once you get them into a topic, then it's half the job, whereas if they think, 'I can't do this, I don't want to do that', then you've kind of lost them before you've even started.

R: Just finally about the course here…what have you learned from the maths modules here about how to choose good examples in maths lessons?

S: Probably more…the theories, Liebeck and that kind of thing, and more to do with the games and things rather than specific examples, I would say…

R: More about the style of the lesson?
S: Yeah, I've picked up more with ideas for lessons than specific examples, and I think that was more down to reading, and there are some, but it's not the bit that's stuck in my mind from the whole course.

R: Do you think, having gone through all the placements and looking forward to next year, you might have benefited from having more input into how to choose specific examples?

S: Yeah, I think so because if you're teaching say, subtraction, where do you start, which numbers should you start with, like from reading, you know you start with this, then go on to that, but its specifically for people who struggle more with maths, then they might think, where do I actually start with this topic, and even within maths you'll have areas that you have difficulty with and when I looked at time, I thought, 'Oh, where do I actually start with time? What do I teach first, to lead on?' and you have to really think it through yourself before, so that might help, sort of like, sequencing.

R: That's the end of the questions, anything else that comes to mind about maths examples you want to chip in?

S: I just don't like worksheets…I think there is a place for them sometimes, and I would use them at times because they can process knowledge if you've been doing a lot of practical work, but I prefer to use a lot of games and things, especially I've found that with low attainers you do need to use more games because you need to get them up and do it because they struggle and they don't want to do it, and I think even with high attainers you should still get them up and get them excited about maths because I always really enjoyed maths at school, which is where my sort of like passion started, and I know that everyone else hates it, which is why I get a lot of stick for it, and erm…it's just one of those things I really enjoy and I want other people to see that it can actually be fun, it doesn't have to be really difficult, and like, even though it's difficult, it's a challenge and once you get through it then you…I don't know, you feel more self-achievement, don't you?

R: Okay, thank you.

**Appendix 8c**

**Interview transcript – ED13 'Dawn'**

R: Right, 5th of June, first interview of the day, so first of all, did you enjoy your final placement?

S: Er, yes and no. The erm…I got on really well with the children and it was a school where they do the Creative Curriculum which was really, really interesting, so I enjoyed the teaching that I did very much, but unfortunately I didn't get on very well with my class teacher, so…I found her very unsupportive.

R: What year group were they?

S: Four.

R: What were the class like?

S: They were nice…sort of mixed social background, some of them quite difficult social backgrounds, erm…a couple of what I might term characters in the class, but basically lovely children.

R: Have you got a particularly good memory from that placement? One thing that stands out as being memorable?

S: Erm…I think just how well I got on with the children, erm…you know, some classes you really click with, and that was one of them, so yeah.

R: Let's talk about planning maths lessons, so start from scratch, how would you go about planning a maths lesson? Suppose you had to teach something next week, you knew the topic, you knew how well the children were at that topic, what would you do?

S: Erm…first of all I would…you say I know the topic, but first of all I would kind of gen up on the topic because it's many years since I did maths as a child, erm…and some things I find hard are taught differently, for example, sort of…the way you can't borrow one and pay it back anymore, so…I have to double check the way things are done now…so I would do that to start with…and then I would…think about trying to make it as kind of, creative and hands-on, if possible, and…erm…I was able to do that in this placement but in my last placement they had one of the set schemes and they followed that very particularly and didn't want anything to, sort of, interfere…

R: Which scheme was that?

S: Erm…the Heinemann one? So, yeah, I would look for a creative way.

R: So how do you then go about deciding what to do in each part of the lesson?

S: Erm…I would…you know, do a wall map depending on what the focus was. With class I was with, they were focusing on trying to get their multiplication tables up to speed, so one of the things I did quite a lot with them was sort of, follow me cards, and they quite like the element about…'cause they like it when we, sort of, time it each time to see how quick they could be…erm..then obviously the main body of the lesson…one of the things I found is that I tend to talk too much, so I would try consciously for future planning to cut down the amount of talking I do and get on to the practical activities, and then try to remember to do a kind of mop-up at the end.

R: What resources have you used to help you plan your lessons?

S: Erm…to help me plan my lessons I have used the erm…I think it's called Teachers' Resources website, the one that teachers put resources on…I've got a selection of you know, erm…those things I wouldn't use…I'd take them and adapt them…they are quite useful as a starting point.

R: Have you used the interactive planning tool on the online framework?

S: I have used…I'm not sure if I've used that bit of it, I have used that and I've used…erm…a lot of the interactive teaching resources that are on there, and found some of those really useful.

R: But not the planning tool itself?

S: No, I haven't, no.

R: Did the school have a scheme that you could draw on?

S: Yes, they used Abacus, but…it…

R: Did it influence your planning at all?

S: Erm…one of the things that happened while I was there was that their maths co-ordinator sort of said, 'oh, well, we're not meeting this…sort of, certain targets', and she was trying to get the teachers to go back to using the Abacus scheme more logically than they had been doing, but I didn't find it, 'cause I had a stop set in maths…find it stretched them.

R: So you didn't use it?

S: I used it…I used bits of it.

R: Thinking now about the actual examples you use in maths lessons…to what extent do you think the choice of examples is important for learning?

S: Can you describe examples…what do you mean by examples?

R: If you were going to teach subtraction, say, and you start off with 20 subtract 9…

S: Yep...

R: There's obviously going to be borrowing going on or use of a number line, so do you think the order of the examples is important?

S: Yes, I think it would be, I think you need to, sort of, start simple and progress up to , you know, more complex issues.

R: So if you are using a scheme or a website, how much consideration do you give to the examples they suggest, as to whether to use them or not? How much do you change them, or make your own up?

S: I would erm...I would sort of change them or make my own up, I...because erm...as I say, because I'm...not that I'm bad at maths, but because it's kind of done differently, I'm always very...erm, I go through examples very carefully and make sure that I understand them in order to be able to explain them.

R: Have you ever changed the examples they give for ones that you think are better?

S: Yes.

R: Can you think of examples where you've done that?

S: Erm...no, not off the top of my head!

R: But what sort of reason might you think for changing an example? What might it be that you don't like about the ones given that would make you change them?

S: I think...sometimes I have probably substituted simpler ones to start with and then used...erm...you know, the ones that are given. Maybe just sort of things that fit in better with what point I'm trying to get across.

R: So when you decide on which questions to use to demonstrate something to the children when you're teaching the direct whole class bit, what factors do you think of when you are deciding which examples to use?

S: Obviously I've gone through them first to see that I can do them...erm...what else...I think about ones that might present problems for them so that, you know, I can go through it and explain those problems before they arise sometimes, or as once happened, they'll start something and then a question will crop up and then I'll go back and go through perhaps a different example.

R: And if you design a worksheet for children to use, either whole class or groups, how would you decide on the sequence of examples on the sheet?

S: I would again start with perhaps some simpler ones and move onto more complex ones, but then I would probably put a mixture, I wouldn't just go from easy to hard, I would start easy and then go through...

R: Are there any theories or models or any literature you've come across that gives guidance about choosing good examples?

S: I don't think so!!

R: If you knew that there was some sort of theory-based guidance on choosing examples, is it something you'd be interested in using?

S: Yes, yeah, definitely. Anything that would help my maths, but that, I think, is something that would be, you know, really, really useful.

R: Maths subject knowledge, let's talk about that. How would you say you feel about your level of maths knowledge for teaching?

S: I'm much more confident in it now, when I started the course, I hadn't used maths apart from like, the everyday maths that you use as an adult in terms of sort of, working out your family budget...erm...that kind of thing, I hadn't used maths since doing my maths 'O' level over 20 years previously, and it would have been a bit of a standing joke that my maths isn't very good, but in fact it is much better than I thought it was...erm...but it was very, very rusty so things like space and shape, things that you don't use...I'm married to a person who measures how much paint we need for the kitchen and how many tiles we need and everything, so I've not done that...so it was actually better than I thought it was, even things like algebra, which is probably my weakest subject, is okay, as long as I'm kind of, you know, do the preparation, it's definitely kind of, only 'O'-level, it's not any higher!

R: But for primary teaching?

S: Oh it's adequate for that, yes.

R: How much do you think your level of knowledge helps you choose examples?

S: Erm...I...to be honest, before we had this conversation I'd never really given it that much thought, I put thought into it, primarily because I have to make sure that I'm not going to confuse the children by doing something in a way that they are not used to, so that's why I put thought into the examples I choose rather than anything else.

R: So it's your preparation rather than your level of knowledge?

S: Yes.

R: If you have to teach a topic that you're a bit shaky on, perhaps, fractions or something, what would you do to go about improving your knowledge on that before you teach it?

S: I would work through...you know, some examples first...erm...one of the things I find difficult is if I'm doing supply teaching, turning up in the classroom in the morning and then saying 'Right, you've got to teach...' ...it was probability this week and then I haven't had a chance to work through some examples and then I have to go with the examples that I'm given and I find that quite uncomfortable.

R: Do you use particular books to help you brush up on topics?

S: I have got just the...erm...the kind of revision guides that you would have at Key Stage 2 and the sort of Key Stage 3 ones...erm...I've got those and I've used those quite a lot, I've also got children, older children who are 12, 18 and 19 and that has been quite useful because I've used a lot of their old stuff as well.

R: Do you use things like the Haylock text, or...?

S: Erm...it's not...I, yeah, I do use them, I use them more for assignments to be fair than for day to day teaching, erm...there are a couple of other books, the kind that deal more with diagnostic problems, and I do tend to use those, but Haylock is not the most friendly book!

R: Do you think if your maths knowledge improved, through teaching and revising and brushing up on things, you would be able to choose better examples?

S: Yes. Yeah, I think it would...erm...yeah, I mean at the moment it's a case, I think, of you know, sort of working out what you need to get by, I would like to have a sort of greater depth of knowledge, a sort of greater comfort zone...

R: Which will come with practice...

S: So things about just generally teaching and learning maths, you've obviously been alongside experienced teachers on placement, what have you learned from seeing them and how they plan and teach maths?

R: It's been really interesting over the three placements in different schools, and being with a variety of different teachers, erm...one teacher I was with in my first year, third placement in my first year, very experienced teacher but actually taught the maths with the scheme book open on the desk in front of her, and taught exactly from the book, which surprised me because she said she'd been teaching for more than 20 years and yet she obviously still felt the need to do that. Then I've seen other teachers who don't seem to do any planning, they'll pull examples out of their head and do them on the board there and then. I can't do that because I'd have to work out the answer first!

R: When they do that, do you think 'Oh that was a good example, I wonder where that came from?' Does it impress you the fact that they can do good examples just like that?

S: Yeah, yeah.

R: Thinking about the modules here, the maths modules, have you learnt anything from the modules here about how to choose good examples in maths?

S: I don't think there have been any specifically about that, about choosing examples…erm…they've been much more…erm…sort of generic, if you like, I don't think we've ever got down to 'this would be a really good example' or 'this would be a really good way to choose examples in this particular topic'.

R: So it's more about delivering the Strategy than how to teach specific concepts?

S: Yeah, yeah…

R: So do you think you would have benefited from having more guidance as part of the course on how to choose good examples?

S: Yeah, yes and I think, you know, one of the things we've talked about as groups of students is that maths in particular would benefit from being more…you know…more kind of 'nitty gritty'.

R: Okay, that was the last question, as we've talked is there anything else that has come to mind about how you plan or teach maths ?

S: Well, now I feel really bad, because I've never actually thought about, particularly about the examples I've chosen or why I've chosen them, it's much a case of, okay this is what I want and if I do that example then that shows so-and-so…and I can do it!

R: Okay, thank you.

**Appendix 8d**

**Interview transcript – ED26 'Rachael'**

R: Did you enjoy your final school placement?

S: Yes, I really did, but it was a bit of a challenge.

R: So what year was it?

S: I was in Year 3 at Wxxxxx Primary School, with a lot of EAL children and I've never been given that before, so it was a good experience really, and there were quite a lot of behaviour issues so I was just trying to think of different ways to overcome them really, different strategies.

R: And what were they like as a class?

S: Well I really liked teaching them actually, I really enjoyed it, there were big characters in the class, really funny. It was nice, yeah. It was more exciting than other schools I've been in because of the characters.

R: What were they like at maths?

S: If you were to level them, they'd be quite low, a lot of them were still on P levels, even though they were in Year 3, so their ability ranged a lot and then there was one child that was on the gifted and talented register for maths, so there was a big gap between them.

R: What level was that child?

S: He was working at Year 5 level, yeah, there was a big jump between them so I was always, like, the lower ability ones would have to do more of a game activity or something to keep them but they still need to meet the same thing.

R: Have you got a particularly good memory from that placement, either maths or anything else?

S: Geography, geography is my specialism and I really liked it and a lot of children brought in things from home, and it was usually about the topic we were doing, which showed that they were really learning and enjoying it, so that was nice.

R: Anything in maths that stands out?

S: Because they found it so difficult, when they achieved something they were so proud of their work and especially the lower ability children, if they got a certificate for achieving things they were chuffed with themselves the rest of the day.

R: How do you go about planning a maths lesson?

S: I would first of all look at the 'I can' statement and think about the ability of the children and make sure that the different abilities met the 'I can' statements, and then I'd think about the task and I'd make sure it was differentiated, so in the class that I've just done it was usually differentiated three different ways, and first of all there'd be a starter and I'd think about doing something on the interactive whiteboard to engage the children, it might be something they've already done, or just something on multiples, then erm… then for the main bit I'd do a whole class introduction which might involve them using whiteboards or number fans and then send them off to do individual tasks, I'd make sure I'd planned all that in my lesson plan.

R: Which resources would you use to help you do the planning?

S: Primary Framework, the new Strategy, and just check on the strands and blocks, and I think I'd ask other people, at this school especially a lot of the other teachers had good ideas for EAL children so I used them as a resource.

R: Did the school have a scheme?

S: No they didn't.

R: So would you use websites to get material, or books?

S: Yeah, I use the Mathszone website quite a lot, I thought that was really good for games and even just for getting some ideas really off that, and the BBC Bitesize website, that was really good.

R: For Year 3?

S: Yeah, so I could learn about it, I don't think it's called Bitesize, it's a primary section or something, and they'd all done it at home as well, quite a few of them, so they liked it as well.

R: Have you used the interactive planning tool that's on the Framework?

S: Oh, I'm not sure, I don't think so.

R: If the school didn't have a scheme as such, what did they do to get their maths planning? Was it all from the Framework?

S: Well, to be honest with you, it's a bit of a controversial thing, I know that quite a few of the teachers were using a scheme they weren't supposed to, and they'd take the worksheets from it, but I can't remember the name. It just literally told you task 1 and task 2 and that. And when I got there a lot of people said they weren't really supposed to be doing that 'cause I had asked what kind of planning they did and it kind of got brought up that they shouldn't be doing it and erm… so they had to stop doing it and then they just had to use the Framework. It was all a bit of a muddle really, but most of them just took it from the Framework.

R: So that's what you did for your planning?

S: Yeah, I just took it from there, yeah.

R: When you are teaching and using examples to show the children and to help you explain, how important do you think those examples are?

S: Yeah, really important, especially in this school I found it... because of the abilities, I found it really important and I observed that the teacher had said that some examples were for a particular group, so they'd be a bit more aware that that question was for them, and a lot of the time, they'd do a bit of the examples and the lower ability group would be sent off to do their own with the teaching assistant or another teacher.

R: When you looked at the Framework and decided on a topic, how did you choose your examples?

S: A lot of the time, so if I was doing something for the interactive whiteboard I'd literally just go to the Mathszone and go through them, and some of them were either not suitable or too difficult and then I'd choose ones that I thought were good, if I was just doing examples that I was just drawing on the board then I might think about them before and jot them down, and then just do it as I was teaching, so I would really just have concrete examples to use and see what the children picked up on and if they were struggling I'd change my examples.

R: So it was for ideas of examples rather than definite ones?

S: Yes.

R: Alright, so when you decide on an example to put up for the children to look at, is it usually a random choice if it's a number problem? Do you make the numbers up as you go along?

S: Sometimes I will and then other times, we did word problems and I would write it out neat to make sure I had it in front of me, sort of on a post-it note, a few examples that I knew had been worked out right, just to make sure that... so if I wanted to do it so that it wasn't correct so that I could put it up, I'd always have an example that worked but it was incorrect so that they could see that it didn't work, so I wasn't doing it on the spot, so it was pre-planned.

R: Did you use worksheets at all for the children?

S: Erm... yeah, sometimes, but I tried not to, a lot of the time I tried to do it so they had games and I made resources and things so that, like we did fractions and you could actually match fractions and make their own fractions, because I found that when they did worksheets their behaviour was even worse than when they were actually involved in it, so I tried as much as possible not to use them.

R: When you did use a worksheet, how do you decide the sequence of the examples to put on the sheet?

S: I tried, especially with the abilities, to try and build up the questions, each sheet would be differentiated for the different ability groups and then I'd start with easier questions and build up to difficult ones and then maybe at the end put an easier one again, so they weren't aware almost of how the order would go, like from easier to harder.

R: So usually easy to difficult, but a bit mixed up as well?

S: Yeah.

R: And where do you get the ideas for those questions from?

S: I usually make them up, if I was really struggling I'd look on the internet but usually I just sit there and make them up.

R: If you have used any off the internet or anywhere else you've found questions or examples to use, do you use them as they are or do you change them if you aren't happy with them for any reason?

S: Yeah, I usually change it, yeah, 'cause I like to think as well, if I'm going to make my own worksheet, then it is my own question, so even though I might take the idea because it might work very well, I still want to change it to make it my own.

R: So are you changing it just so you can say it is your example, or are you changing it for any other reason?

S: Ermm… I probably change it so I can say it's my own, but also, I've had, like, previous experience of like, I've used it off the internet and the children say they've done this and then you think you've got a whole thing here that they've done, so I think that's another thing that it's a safety net really and if I change it, that can't happen!
R: So when you decide to choose your examples to put up and show and teach the children, what things do you consider before you use the examples?

S: I probably… I'd introduce it and then I'd, if it was in the main task and it was something I'd covered before, I'd get them to tell me what we've done already in the week, and then if it's something that they are struggling with, so from the previous lesson I'm aware that they can't do something, then I'd choose that as a focus for an example for the teaching bit, and then say we are going to focus on this because it's what they have been finding difficult, and then do an example to see… and talk it through and see if it helps them to understand it more.

R: Have you ever asked the children to make up their own examples?

S: Erm… only if they finish, I would then see if they can think of a few of their own questions and swap them with a friend, and sometimes I find that works really well if they've got the concept and other times it doesn't.

R: Thinking about the course here, have you come across theories or models or ideas that people have written about that have helped you with your maths planning?

S: Well, for my professional studies essay I did all about the 6 Rs for mental and oral starters, and before I didn't have a clue about what they were at all.

R: Tell me something about those.

S: Well they stand for different things, there's rehearse, recall... there's six different ones and it's from the Primary Framework and I didn't know anything about it, and so know I realise that when I'm doing it, I think is this a rehearse or a recall to try and make sure it's different ones, but before I wouldn't really have considered it, I just thought it was a mental and oral starter. It's in the Primary Framework, in the library section, and I didn't really know about it and I did it for my professional studies, I was doing it on mental and oral starters and my tutor said there's this thing called the 6 Rs, so I researched it and I know use it when I'm teaching and I find myself thinking about it without realising it which I thought was quite good. But I think with lots of things when you do reading you kind of just do it for the essay, and you just think all this fits for the essay and you don't think about it. Apart that there is VAK which is like if it's visual, or do something that's auditory or kinaesthetic.

R: Have you used that in your maths planning?

S: Yeah I always do, and especially this time I was trying to do visual all the time for the EAL children, so erm... and then ELPS as well, we do that in maths, about whether they are getting experience, and the language and the pictures and the symbols, so that was really important, like when I was trying to make things I was giving them the experience, we were doing about money so they made their own shop.

R: So you actually thought about ELPS as you were planning the activities?
S: Yeah, and they had erm... a discussion about it, a meeting while I was at the school and saying about how important it is to try and plan in your ELPS and to think about it and the VAK as well because of the EAL children, so it was quite good really, they were saying it's important to have experience... I think because it's so focused on... there were so many children with EAL that maybe try and think about it or about visual learning really.

R: Have you heard of the Knowledge Quartet?

S: No.

R: There's a lesson plan here about what happens when you swap the digits round in a number, as a starter, so you've got for example, 45 becomes 54, I wondered why you would have picked those particular numbers?

S: Erm... there probably wasn't any reason, I think I just did 2-digit numbers and I could have chosen any 2-digit number really.

R: Well, if it's a 2-digit number where the digits differ by 1, if you swap them round and then take them away, you get 9.

S: Ah... no, that wasn't the reason!

R: For Year 3, that probably wasn't... and if the 2-digit number has digits that differ by 2, when you swap and subtract you get 2 x 9, or 18. But yours was just about swapping the tens and units over?

S: Yeah, I just sat there and thought about 2-digit numbers.

R: So did you do any like 20 or 50, with a zero in?

S: No, they were always different digits but not zero.

R: Here's one where you mention a low 3-digit number. Did you mean a number that is just over 100?

S: Yeah, we were doing it just over the 100 mark, and then I can't remember if we progressed to doing more or not, but I just kept it low because of their ability.

R: So did it matter, for example, that you've got two 1s in it, or any double digits?

S: No, no reason, just random!

R: Going back then to the theories you've discovered, do any of those help you choose good examples, rather than teach you how to teach a good lesson?

S: For me, I would say that if I'm thinking about them when I was planning I would focus more, and that that's the way I should teach my lessons, and that I should teach it using visual resources, or that I should make sure the lesson appeals to auditory learners, and then if I did think about it, I would maybe think I need to make sure there's experience in there, and the example I'd use was about an experience, but I would say if I was thinking about it, I'd probably focus more on the lesson rather than the examples.

R: Even though you said the examples are really important?

S: I think examples are really important but I wouldn't necessarily consider the theory when I was choosing the examples, but I might do for when I do my planning, but not when I was putting examples on the board, I think I'd be more just making them up in my head.

R: Could that be because there isn't any good theory about examples?

S: Maybe, but I think purely that when you're teaching, I don't really consider the theory, and I think maybe when you're new and you're writing your essays and it's all there in your head and you consider it without realizing but I wouldn't really do something because of what the theory says.

R: If there had been a session in one of the modules that said this is a theory on how to choose good examples, you might have used it in your essays, but when you are in school, do you think it would have made a difference if you had been shown some theory on choosing good examples?

S: Yeah, then I would use it, if I knew that it was specific on examples and I wasn't really sure what examples I should use to teach different things, then I would use it, because I find myself remembering more, specially from the maths seminars, like this time I remember someone using coordinates and then I was thinking when I was doing it, I remembered that happening in maths and I'd look and see what we did, and so then I would use it but not necessarily use theories, more what I've seen in lectures.

R: Let's talk about your maths knowledge – how do you feel about your level of maths knowledge?

S: I don't feel very confident, maths would be like my weakest subject, when I was at school I never really enjoyed maths.

R: But given that you have to teach it, and you could be given any primary class to teach, would you feel confident enough to teach any primary class?

S: Yeah, I suppose, and I'd feel more confident… when I did my GCSEs I had extra tutoring from a maths person, it was more a confidence thing, I think I thought I just can't do this, although I probably can but I sit there thinking I can't do it, then I found that really helped, and when I was with Year 6 I asked the teacher a lot, how do we do this, before I actually teach it.
R: So how did your GCSE turn out in the end?

S: I got a B so I was pleased.

R: If you have to teach something you are not so sure about, where would you go to find what you need to know to be able to teach it better?

S: I've got erm.. I've got this book that I use, and Haylock as well, which is quite useful, especially the back bit, where it's got like in each chapter it's got the vocab for it, 'cause when I got to Year 6 there were some things that I wasn't really sure about, but I'd say that book is quite confusing on telling you how to teach it, really, it's quite… and if I'm not sure, I still look up… I've still got GCSE revision books and that, so I look at that and see the theory, although I think it might be a bit ahead, just to see really, I use the internet as well, a few different places.

R: So you are happy just to go to things to brush up your knowledge if you need to.

S: Yeah, just so that when I'm teaching it I feel like I actually know, rather than being a bit unsure or that they know more than me.

R: What do you think maths knowledge includes?

S: Ermm... my ability I suppose, in maths, and how much I know about the subjects, and how good I am at it, I think.

R: So the actual maths itself, like can you do fractions, can you do probability?

S: Yeah, the subject knowledge.

R: Do you think that's all maths knowledge includes?

S: That's all my maths knowledge includes!

R: If you include all the topics, fractions, algebra, all of that... that tells you what to teach, but what about the set of knowledge about how to teach? If there are those two sorts of knowledge, do you still feel as confident about your maths knowledge? Are you confident in both areas, both the topics and how to teach them?

S: I think in a way, they kind of link because if I'm not feeling very confident with a topic, like long division or something, then I wouldn't be very confident in teaching it... I did long multiplication in Year 6 and they do it in such a different way to how I did it when I was at school, I found that I would have had to look at it even if I did feel confident about it, so in a way I was quite glad I did look it up.

R: So even if you are confident in one of the topics, you might still have to look up how to teach it?

S: Yes, just because it's different to how I did it.
R: You said you'd look up in say, Haylock for how to do the maths, where might you go to look up how to teach the topic?

S: Now I found that really difficult, because there's not really... I wasn't really sure where to look, and then a lot of places do it so differently, specially on different internet sites, so mainly I'd ask the school teacher, and I did one lesson where I did it and it was addition and I did it going down and they did it going across, and then I found I would have to ask how they should be doing it at that school, because each school seems to do it slightly different.

R: Do you use the Framework for how to teach things?

S: No, I never really looked at it in that way, you only really think of it for planning.

R: I f you have those two types of knowledge, the topics and how to teach them, do you think having good knowledge, of either sort, helps you choose good examples?

S: Yeah, I reckon if you're more aware of the subject knowledge and how to teach it, and if you've got that step ahead, then you can choose examples.

R: Which do you think is more important, knowing the topic or knowing how to teach it?

S: Maybe the how to teach it because you almost have... to know how to teach it, you're teaching the children and they need to know it, and I think you know the topic by knowing how to teach it and you don't need to have really good knowledge of that because you must have basic knowledge to know to teach it, if that makes sense.

R: So for a good example, if you know the topic, say you know all about fractions, does that mean you can give a good example to children about how to add fractions?

S: Yeah, I'd say probably, but then I suppose you could have somebody who was really good at maths, and they might not give a suitable ability example, or they might not be aware of an example that would help children understand it, if you give them an example that is almost done for them, or not done enough, that's one thing I find when I'm doing examples is, that they're not actually getting anything from it because it might already be giving them a bit of the answer too much.

R: So are you maybe using examples that aren't very good ones because the children aren't learning what you want them to from them?

S: Yeah, a good example would be one where they learn quickly what it is you are trying to teach them.

R: Do those good examples come more from the knowledge of the topic or more from the knowledge about how to teach it?

S: A bit of both probably, but I think more from the knowledge of how to teach it because you could have excellent subject knowledge but not the skills to transfer it.

R: What have you learned from observing classroom teachers about how to teach maths?

S: This time I realised how the teacher would address the different ability groups, and that worked really well, and then gave each of them their own examples and then sent them off to do their work so they weren't all just sat there thinking this wasn't for them, and when I was in Year 6, the teacher there was really good at doing the examples and she would just do different ways of doing it so she would just get then up and involve them or have only a few children involved in doing the examples so the rest could see it, and even the ways of doing long multiplication, when she would just do them on the board, her language was very clear and sometimes I think when I do it, it might not always be as clear and you can tell because they just sit there, you know. But then one teacher I saw before, they said were doing fractions today, and just gave out the worksheet and didn't do much of an example and then they'd go to their table and all be a bit lost, so that's a good reason to do the whole class teaching with examples before so that you can pick them up.

R: Thinking back to the course here, the maths modules, can you remember anything in any of the sessions about how to pick good examples?

S: I would there were quite a lot of seminars on more… the topic, not the knowledge of how to do it, and then also sometimes there were examples, one lecturer did quite a few, she shared lots of resources which were good, but I wouldn't really say that we've had it on number as much, so I don't really feel that I would know what is a good example from uni, I've just done it from experience, so I don't really think there was anything on choosing good examples.

R: Thank you.

**Appendix 8e**

**Interview transcript – ED37 'Andy'**

R: Did you enjoy your final school placement?

S: Yes, it went very well, I enjoyed it.

R: Tell me a bit about it.

S: Erm… well it was lots of new experience with an older Key Stage group so… I worked with Year 5, most of my practice had been with Key Stage 1, so it was a different experience, especially with planning, preparing, behaviour management, the whole thing was different, so again, a lot of experience in that area.

R: Nice class?

S: Yeah, very nice class, yeah, very boy-heavy, it was quite challenging but it was good.

R: Have you got a particularly good memory from the placement?

S: Yep, I tried to use as many different resources as possible so I tried to make it interactive and very visual so I used a lot of the interactive whiteboard, for my teaching time as well, and I focused a lot of my mental oral starters on the interactive whiteboard and tried to use it at the end of the lesson as well, which I hadn't done much of before, and trying out programs and software.

R: Any particular incident you remember?

S: Oh yeah, I noticed there was one child, and this was quite rewarding as well, there was this one child who very rarely got involved in conversations, very rarely asked questions, very rarely put his hand up, but using the interactive whiteboard for the maths activities, he came out of his shell, started answering questions, got more involved, came up to the board and had a go which you'd rarely see from him normally… not because of my lessons particularly, but because it was interactive and I enjoyed that.

R: Let's talk about planning, so how do you go about planning a maths lesson?

S: I do it very similar to the rest of the way I plan, start with a piece of paper, a broad outline, sort of a ladder, stair thing, like KT taught us, so I know what I'm teaching first and have an overview, then I have a rough plan and then I move on from my rough plan to get a finalized weekly plan. I keep my rough plan, so it's kind of a medium term plan but not, but I have an overview for the 4-week block, erm… I did one week at a time, weekly, and plan that in detail in a sort of medium term plan format.

R: So what do you do for individual lessons?

S: Each lesson I have an individual lesson plan and I have key focuses such as what the objectives are, what my aims and outcomes are going to be,or hopefully what the outcomes will be, and what resource I'm going to use, what groupings I'm going to have, who is going to be working with who, staffing wise, basically as much detail in the organization as possible on there, so it's mostly classroom organization.

R: Where do you get your objectives from?

S: I get them from the framework online, it's quite new to me, the new framework, so I found it quite challenging but when you get used to it it's quite straightforward, so I get on there, I work what the blocks are and what block you're going to be in, what letter, so is it A, B or C, and I find it through there, I take my objectives from there.

R: What resources do you use to help you plan the maths lesson?

S: Definitely the internet, that's the most important one for me, so the framework to start with for the planning, then move on to the links they've got on there for the resources, to general internet searches for activities, games, I use Primary Resources and then lots of other sites that I've got written down somewhere but similar to that sort of site, with lots of different areas you can choose from.

R: What about books or schemes?

S: Yes, I go in the library and use the '100 maths lessons' and the renewed framework book, they were very helpful 'cause they have lessons in there for you which you can adapt.

R: Do you ever use other school staff as a resource?

S: Yep, specially the class teacher, 'cause she'd taught it all before and got ideas and things and tried to change it that way.

R: What about Haylock?

S: Haylock's book is very good, I benefited from reading it in the first year but I haven't used it since.

R: On the framework, there's an interactive planning tool, did you use that?

S: No I don't think so, I've just used it, like, the page. I've used lots of resources on there, the links, especially the interactive links that they give you, so they give you grids and different things like that which was quite helpful.

R: Did the school have its own scheme?

S: They used the renewed framework online, however they mixed it with Abacus, so they still used Abacus but they then referred back to the framework just to make sure that they are meeting the criteria and the objectives. Tricky, hard, but it works, using the Abacus cards for activities to support the framework objectives.

R: And that influenced how you had to plan?

S: Well, yeah, I went about it the same way the class teacher would, so I used the Abacus cards for activities but they had to be altered quite a lot in order to meet the objectives on the framework.

R: When you have your activities and the questions and examples you'll use, do you think it's important which examples you choose when you're teaching maths?

S: Erm... well, yes and no, I would say they are important but as long as they meet the criteria and objectives that you're trying to teach, I think that's the main key, I mean, I would order them step by step so start with the simple things, build up so they understand what they're learning.

R: If you use a scheme like Abacus and you change it, why are you changing it? What is it about the scheme that isn't right for what you are trying to teach, and how do you change it?

S: I think specially Abacus, for Year 5 it was just too easy, it wasn't challenging, all the tasks were the same, differentiation wise not very much differentiation...

R: So the Year 5 book wasn't right for Year 5 children?

S: In my opinion, yeah. Just the cards, like for one week, the objective was to add and take away decimal points and the cards were actually very simple and the higher ability needed challenging.

R: So when you change them, do you make up your own version or find something from somewhere else?

S: I like to try my own versions but I often get help from the internet or website.

R: When you change it yourself, how do you decide what to do?

S: I'd still use the objectives online, I wouldn't change them, and I'd still use the activities and questions I had as a basis but I'd try and make it more challenging or difficult for the higher ability, which I needed to do for this class.

R: When you do your teaching input, you have the child sat looking at the whiteboard, you are going to put up some decimal calculations for them to see, so how do you decide which ones to use?

S: I'd start with a very simple one, very easy, straightforward.

R: Such as?

S: Such as… erm, 5.1 + 5.2, or even easier probably, 2.3 + 2.4 or something, and show… line the points up, put it in the right column having set the columns out, then move onto a harder one and a harder one, possibly 3 points, like 5.22, more decimals.

R: Then when you give the children work to do, either in a book or a worksheet, how do you decide on the examples for that?

S: I'd go… I'd first of all judge it yourself first, will they be able to do this based on the last lesson, by marking their work and what they can do already, and then obviously as they do it as well I watch carefully then it's quite simple, it's easy to move them on, so I'm trying to do it do they can do it without needing too much help but with some challenge, I don't the lesson with lots of hands up all the time, but enough to keep them challenged.

R: Did you use worksheets?

S: I started off doing it but I tried to make it more interactive and creative, so I used it less, I used the interactive whiteboard and mini-whiteboards, that sort of thing.

R: If you had a series of questions, would they tend to go easy to more difficult?

S: Yeah.

R: You wouldn't mix them up?

S: Not to start with, no. Probably second or third lesson in I'd change them, do word problems or a bit more challenge.

R: Thinking about the course now, what theories or models have you come across about how to teach maths?

S: There's been an awful lot of strategies, like how to start simple and work your way up, start in small steps and build your way up, we done a lot on challenging the more able, and making sure they are challenged, not just are they doing the work alright, we done an awful lot on making sure your activities are differentiated very well, not just one, two, three ways, there might even be four, five ways for other abilities.

R: So lots of classroom strategies, what about theoretical models or authors you've been told to read?

S: Haylock definitely, one of them, an awful lot, I've done… through assignments, I can't think of names specifically, I think Hayes is probably one of them which I've gone through which is quite good, but from assignments an awful lot of reading to build your own general knowledge and your own strategies and ways of doing things, 'cause our assignments have been very much theory-based, but I can't think of any names particularly.
R: What about ELPS?

S: ELPS, yeah! The big one! An awful lot of ELPS.

R: Does ELPS help you with your planning for maths?

S: It didn't used to but yes, definitely now.

R: So when you plan, you actually consciously think about the bits of ELPS?

S: Yeah, what resources are going to be visual, what resources I'm going to use, what language I will use, like, what mathematical words am I using… I wouldn't say I use the word ELPS, but I use that idea.

R: What about VAK?

S: Just natural things really, so like, the interactive whiteboard is visual, and if they touch it, and sound can come out of it, and what resources we can have on the carpet.

R: Any others?

S: No, they are the main two.

R: Have you heard of the Knowledge Quartet?

S: No.

R: If you knew there was a good theory or book about how to choose good examples, is it something you would be interested in?

S: Yeah, yeah.

R: Does that suggest you maybe haven't had enough of that on the course?

S: Yeah, possibly, 'cause I think when we plan and we make questions up, it's often the luck of the draw, it's pretty random, and questions I think are difficult to them may not be so hard and again, what I think is easy may not be so easy for them, so yeah, definitely.

R: Okay, how confident do you feel about your maths knowledge?

S: Erm… three years in, much more confident, erm… however, I do still find I have to go away and research areas before I teach them, where I don't know them just like that, but some areas, yeah, so if I do fractions I have to go away and do a bit, there's so many different things in fractions so that's definitely one of them, but once you know it, you know it and then if you don't teach it for along time you have to go back and check it again.
R: So apart from fractions… you think you're knowledge is enough to teach right across primary?

S: I'd say generally yes, but I still have to go away, I still have to go and research and check.

R: When you did GCSE maths, would you have said your knowledge then was good enough to teach primary?

S: No… er… maybe! It depends again what area, I would say more my A-levels, although I didn't do it properly, you had to do it throughout your studies, so I'd say more then.

R: So for things like fractions, if you have to go and research it, where would you go to?

S: I've got old books from GCSE revision which are really good, just pages and pages, like a study guide, I use that heavily, possibly online, I go into searches and searches looking for a really simple website, like BBC Bitesize or something, and possibly look back at old things I've done on fractions.

R: Would you dip back into Haylock?

S: Erm… honestly, no, probably not, 'cause I'd have to read and read it to find it, I'd say it definitely is a good book, it's just been killed a bit.

R: When I said 'maths knowledge', what sort of things did you think that was going to include?

S: Understanding of the things I'm going to be teaching the children, concepts, ways of solving misconceptions that children have, being confident in what you're teaching, making sure you teach it well and right in order to achieve the objectives, so it's the knowledge you're going to be transferring to the children.

R: When you choose your examples for teaching, does it matter if your own ability in that topic isn't so good, as long as you know how to teach it well?

S: I can see that being a really good question to say no, it doesn't matter, but it does really.

R: If you're not so good at fractions but you've read up on how to teach fractions to children…

S: I don't agree with that, so what if a child says they are stuck on this and it's something you're not quite sure of, you then can't solve that or help their knowledge improve if you're still not confident.

R: What about if you are really good at a topic, but you haven't brushed up on the best ways to teach that topic, will that help children?
S: No, same as the other question because you need a good understanding, a good knowledge and then good strategies alongside.

R: So do you think your level of knowledge, of either sort, helps you choose better examples?

S: Yeah.

R: Is there anything you've learned from working alongside experienced teachers about how to plan and teach maths?

S: Obviously each school is different, but generally the overall thing would be start with a really good mental... it doesn't have to be related but I quite like it to be related 'cause it starts it up doesn't it... a very clear... not too long, not too short on the carpet, explanation and ensure the children understand before going away, so those sort of strategies. When they go away, obviously differentiated tasks for children, constantly... well, not constant, but regularly stopping to check they know what they're doing, so it's mostly classroom strategies, you learn them by observing and not telling, then you try to pick them up and use them well, especially using resources, and yeah, that's where you learn it all really, more than in lectures I think.

R: And has it helped your planning to see how other teachers plan?

S: Yeah, and I remember particularly a point... a time in school where when the new framework had literally came out and they started using it, we sat down for a good hour with the teachers going through it together and learning how to use it, they taught us like, the blocks, the links, so yeah, definitely.

R: Back to the course again, do you think you've been given sufficient help in how to choose good examples?

S: I wouldn't say no, but I would say not as much because it's been mainly our own knowledge, more about getting our subject knowledge to top shape to teach, rather than ways of collecting questions, to write resources or anything, so more strategy, classroom stuff than examples, but I wouldn't say no because there's been spots where it wasn't just that.

R: Do you think you could have learned more about how to choose good examples?

S: Oh, yes.

R: Anything else you'd like to add?

S: Again, I'd just say it has to be interactive, I found that's the easiest way to teach it, especially getting them in small groups on the carpet and getting them to use practical objects like beads, abacus, specially the one with the poles up, to show them that when you get to 9 you have to bridge to 10, that sort of thing, specially with younger ones, maths songs are quite good, with the young ones, they worked well.

**Appendix 8f**

**Interview transcript – ED21 'Victor'**

R: Start with some general questions, so did you enjoy your school placement?

S: Yes, I did it was really good.

R: And what year group was it?

S: Year 2 and 3 mixed.

R: And what were they like?

S: Er...yeah, really nice, no...erm, no real issues at all, yeah they had one child that had come from...he was supposed to be in Year 4 but they kept him back a year, but he...he done really well that year, and he was on course for a good level and er...they had one child in there with real...really bad learning difficulties, but erm...there was not really that much support for him which was a shame, 'cause they couldn't afford it because it was only a small school, you know, that old thing again!

R: Have you got a particularly good memory from the placement?

S: Erm...yeah, well, quite a few actually, yeah, well just one, I made a mummy...my specialism is history so I did a full scale mummification on a big dummy and it was really good, they really liked that...you know, insides and everything were pulled out and bits to tug and that sort of thing, and yeah, it was good!

R: Excellent! Right, I'm going to ask you about planning for maths lessons...how do you go about planning for a maths lesson?

S: Well its, just in the...if it's a one-off, then you usually know what children are doing at the moment, at the current time, er...sort of in my head, I have built up a good stock of erm...warm-ups you can do, related to the task, and I know it doesn't have to be totally related, but it's always quite good....if it's a one-off, then that sort of thing, but otherwise, in school placement, it's always been from some kind of framework, that the teacher has given us.

R: So how do you go from that framework?

S: Well, the thing is, from the last school, we used Abacus Evolve, and it was the brand new one, and they pretty much do it all for you, don't they? So it was a case of just looking at that, and then we had the choice of whether we change or adapt it to suit the needs of the children, or we could just stick with it, to be honest, to suit the needs, 'cause we had a mixed group, mixed age group, er...year group class, it was quite useful to do your own thing so it's...we just used that as a guide basically and then just build around it.

R: So which resources have you used to help you plan maths lessons?

S: well, obviously interactive whiteboard...it has a whole...I've used, er...you know the...I think it's on the Smart board, it's not the Promethean, I don't know if you've seen one recently, but there's...we had some new ones put in at that school, and its got like an online gallery...they've got lots of...huge types of maths resources, so you get lots of things from that, like number lines, with like jumping frogs on, and that, quite a lot of those, some CD-ROMs, obviously Abacus, erm...and obviously, you know, practical, er...physical resources like dice and things like that, classroom cubes.

R: And in what ways do you find those things helpful?

S: It's really helpful because you can be a lot more diverse in the way you teach anything, 'cause you're not limited by anything, you've got so many resources to use, you think, 'What do I need today?', and get the children up doing this on the board and they can go away and have a practice physically...like, say the number lines perhaps, just something so they can keep on a task, if they've done a number line activity or something like that on the interactive whiteboard, then they can actually use it and do it as they are working when they go back to their desks, you know what I mean?

R: Yes! Have you used the interactive planning tool from the Strategy framework?

S: Erm...not on the...no, I haven't, no. Oh, sorry, is that the...the brand new one? Erm, yes, yes, we have...sorry! Yeah, we did, yeah!

R: Okay, so how have you used that?

S: Erm...we just, well to be honest I used it a lot more in literacy than I did in numeracy, but er...I mean, 'cause we, in numeracy we did so much from Abacus, it was just a case of checking, 'cause they were new to it at the school, to check doing the Abacus with what was supposed to be going on with the PNS,I mean we used it a lot more in depth for the literacy, but that's no use to you!!

R: So, during placement, how did the school's maths scheme influence your plans?

S: Erm...I did use the old Abacus in my previous stage 2 placement, but I found it a lot easier because of the way they've made it, 'cause they've given examples, haven't they, of lessons and like I say, it just helps jog your memory and you can be a lot more flexible I think, so it does help in that way, there's always an idea there if you're just, sort of struggling, you can always adapt it.

R: To what extent do you think that choice of examples in maths lessons is important?

S: To the children, obviously…very important, because, I mean…bit of a tricky one there…I don't know…I know what you mean, erm…I'm trying to explain it…erm…

R: Okay, let's move on a bit…if you used a scheme like Abacus or something, either for ideas to teach with, or for sheets for children to fill in or for activities, how much time do you spend considering the examples on what's already there, and the sequence they are in?

S: I know what you mean, yeah, erm…there have been times when I've looked at it and thought 'I don't like that', so I'll just do my own…do my own thing whereas I suppose it's probably 60-40 during that last placement because we did have a lot to do, and I'm not saying I didn't want to, you know, do my own thing…for a longer time…yeah, but I'd say 60-40 it was probably that I did it.

R: So sometimes you did change what you saw?

S: Oh yeah, like I said, yeah, probably more often than…you know, 60% I did my own because I'd look at it sometimes and say 'That's not right' or…because it's quite an intense schedule, like its 5 days, one lesson every week, obviously you're not going to get everything done in one lesson, so you sometimes spend half the week doing one lesson.

R: So if you saw a sheet with some examples of calculations, or any other topic, what sort of things make you look at it and think 'That's not right, that's not what I need'? Why would you think about changing them?

S: Erm…from assessment from previous lessons, I think, 'cause you look at it and think, you know, the children aren't there yet, and if some these have done that, then you need to go on and do this, specially in a mixed year class, you sort of, you've got children's…sort of, from one scale to the other, so I think you need to be a bit more adaptable in what you choose. If you see an example, you might think that's ideal for that child but not for the next child, so you just adapt it and differentiate it a bit.

R: So if you do change examples, it's done on ability and matching their levels?

S: Yeah, pretty much, yeah.

R: When you decide on examples you're going to use while you are directly teaching, what sort of things do you consider when you are going to choose your examples?

S: Erm…are they exciting? Do they grab children's attention? I like to get the children involved quite a lot, so I try to get the children involved and do the job at the same time...erm…the other one is they've got to be practical and effective and fun really, you know, they are three things I sort of always go for.

R: Suppose you are doing some work on addition or subtraction, for example, and you are going to choose some examples while you are teaching the whole class to demonstrate something about subtraction. What sort of thing would make an example in subtraction exciting, engaging and fun?

S: Erm…you know, like a shop scenario, that sort of thing…
R: So it's the context it's in, rather than the example itself?

S: Yeah, the context it's in, yeah, yeah…

R: What about the actual numbers you choose? Does it matter what numbers you choose when you are working with examples?

S: No, I think 'cause, I think if you can make it…put it in an exciting context, I think you can do most…what some people would consider, most simplistic, maybe boring things, you know, I'm not saying they are, but erm…I think you can, you know, can teach that effectively in a practical context.

R: So it's the context that decides the examples rather than the numbers themselves?

S: yeah, 'cause I always think you know, rather than just doing some subtractions on the board, if you, sort of, like I said, just for example like a shop context, or something like that, then…I've done that quite a few times before, and it's quite effective really.

R: If you design your own worksheet or activity sheet, how would you decide the sequence of the examples on the sheet? Would there be a sequence, or would it be fairly random?

S: No, there is a…from my own experience, there always tends to be a sequence, it starts off fairly simple and easy, and then it gradually gets a bit more difficult, then towards the bottom there's always some other activity, or once you've finished that there's always some other activity that draws it all together, probably a sort of classic, linear approach really.

R: So again, if you were doing, say subtraction, and…what would you give as an example of an easy one, a medium one and a hard one?

S: Something easy would be something like subtracting multiples of ten, something like that, so its when you take ten, that sort of thing…erm…then probably go over to double…if it was for the age group I was doing at the time, Year 2 and 3, probably like, erm…32 take away…erm…like, 24, something like that, and so it's mixing the numbers up a bit, erm…then the higher, a higher one, well at the time we were doing 3 digits, subtracting multiples of ten from 3 digit numbers, so I'd just go up in levels like that if they already had the experience of doing…had experiences of using those sort of multiplications.

R: Are you aware of theories or any literature or any models that you've read about or been taught about that give guidance about how to choose examples?

S: Erm…not really, no…not that I can remember…it doesn't really come to the fore in my planning, anyway, when I'm in school.

R: If you knew that there was some sort of guidance to help you choose good examples, that was based on some theory, is it something that you think you'd have time to look at and consider?

S: Yeah, definitely, yeah,' cause I'm always looking for other ideas, specially, you know with the internet as well, 'cause it's so quick to do, so I would, definitely.

R: Just some questions about subject knowledge, would you say you feel confident about your level of maths knowledge for teaching?

S: For teaching, yeah, I mean I always struggled with maths to be honest, but when it comes to…I really enjoy it, teaching, and because I spend more time sort of reading and researching it.

R: And how much do you think your level of maths knowledge helps you choose good examples?

S: Yeah, I think sometimes it can be limiting, 'cause sometimes you look at things and think I don't quite understand that fully, so I won't bother teaching…you know, I might leave that today and teach like a starter activity involving it and then, you know, I always think…its just a bit limiting sometimes. I've never really had a problem, it's just, you know…

R: Yes. How do you try to improve your maths subject knowledge ahead of teaching a topic?

S: Well, I always…erm…always resort to Haylock and things like that, refer to literature like that, also a couple of good friends of mine are, you know, really good at maths, it's like their strength so I always just have a discussion with them and they help me out with examples and things like that.

R: And how do you think maths knowledge can help to choose good examples?

S: Quite easy, 'cause I think if you're confident in something, you know a lot about something and there's no…there's no…if you've got any worries about…I think the biggest concern is if somebody poses a question, if a child poses you a question, you don't know about it and you have to keep on saying 'Ah, sorry. I'll try and find out', then if you've got your eyes in the back of your head you can say, 'Give me a second, yeah, I know, I understand what you mean' and you can explain, then that's a real strong point I think.

R: Just some final questions…what do you think you've learned from planning alongside experienced teachers about maths?

S: Erm…the fact that I think, like…erm…it takes a long time to teach children, you know, what we think is a fairly simple idea, you know, I look at subtraction and things like that and even multiplication and things like that…when you go back, into the school and you teach Year 2, and you think, you know, there's some Year 2s and 3s, er sorry, Year 3s and 4s that don't know their second times table, and its just, you got to go back to basics and so in planning for that, the other teachers I think, er…it's really important to just…just take your time basically with it, if you can.

R: So how has it helped you, when you've seen what other teachers do, to plan maths and choose good examples? Is there anything particular in another teacher that you've seen them do, and you've thought 'I'll use that, I guess that's a good way of planning my maths'?

S: Erm…yeah, there's…I'm trying to think what…in stage two, there was, at (name of school) school, 'cause they didn't use things like Abacus…they did have the Abacus, but they didn't have the online, 'cause it wasn't online, the old one, was it? But erm…I can't remember, there was something, sorry Ray, there was…

R: No problem…what have you learned from the maths modules on this course abouthow to choose good examples in maths lessons?

S: Erm, just you know, being adaptable with the children you've got, I think, and I also, I didn't know a lot about, you know, the importance of starters and how it helps lessons to flow…you know, that maybe you teach with a starter and then the main part and then the plenary as well, so that, it really helps you to sort of…you know, it really helps you to sort of teach in that pattern.

R: So that's guidance about the structure of the lesson.

S: Yeah.

R: But if you took, say, the main part of the lesson, have you learned anything from the modules here about how to provide good examples in the main part of the lesson?

S: Yeah, I think we have, yeah.

R: In what sort of way? What do you think has helped?

S: I mean, I think…the way that…I seem to remember, we didn't do much of it this year, but in the first and second year, the way it was, the way the course was structured, as if we were going through the actual, the curriculum, wasn't it, the way…the maths programme, the units, like numbers and shape and things like that…erm…that's helped a lot with the awareness of how it's done in schools, it's a reflection of how it is, isn't it?

R: Do you think that helped more with your subject knowledge, or choosing good examples?

S: I think it helped more my subject knowledge to be honest, yeah, yeah. But with the starters though, that was erm…showing us really good ideas.

R: Do you think you might have benefited from having more guidance on choosing examples as part of the course?

S: Erm…yeah…

R: So would you have got more from a session on how to choose good examples, or were you happy with what you had and when you got into school you were okay with choosing examples?

S: Yeah look, I think there was a lot of emphasis on the starter and the importance of that, you know, which was getting us into activities and that, which was great, there should have maybe been a bit more emphasis on, you know, carrying that sort of element through into the lesson, rather than just sit children down and this is what you'll be doing with them…erm…and the school might have their own, sort of, framework, but it might have been nice to have some…a few more examples, but you know, I was very happy with the course, very happy.

R: Right, that's the end of my questions, anything else you'd like to add about choosing examples in maths lessons?

S: Erm…I think, like I said, the internet is nice, a handy resource with lots of websites…I've used quite a few, I can't remember them by name, but I expect you are familiar with some of them…and that's…I found that really handy, really helpful.

R: If you see something on the internet, a set of questions, examples, things for children to do, do you assume that because it's on there it must be good and therefore use it?

S: No, it all depends on the children you've got, I think, you know, there are so many resources on there, you look and you think that'll be ideal for my class, and actually you can adapt it again, can't you as well, for your own uses. I think it just always comes down to the context of the children you're teaching, really, what they are like and their age group.

R: Okay, thank you

**Appendix 8g**

**<u>Interview transcript – ED24 'Naomi'</u>**

R: Did you enjoy your final school placement?

S: I did, yeah, I did very much.

R: Tell me something about it.

S: It was the age group that I wanted to work with, Year 4, I really want to work with lower Key Stage 2, and really enjoyed the work, it was nice to have two months solid practice where you could go in and you were there every day and it wasn't just bits and pieces that I've seen teaching, you were actually doing it and you got to plan and you could see the progression that children were making.

R: What were the class like?

S: Hard work, but lovely! Very immature in the sense they didn't have much independent thinking skills, needed a lot of mummy cuddling and being told what to do with regards to like, picking a chair up and putting it in a place because they didn't have a chair in their space and things like that, but academically, the range of the children was phenomenal, we had for maths in particular a boy that was gifted and talented from Year 3, and he was a young Year 3 as well, and then you had the other end of the spectrum where you had a boy that was actually Year 5 and he had one to one help the whole time in the class so it was quite a big spectrum, and he had very limited language as well.

R: What were they like for maths?

S: Erm… very diverse. You had the top end of the class, the higher attainers, that could do Year 5 work with a little bit of help, but they could do Year 5 work and then you had the middle who were probably a high Year 4 level and then you had the two lower attaining groups who… I suppose you had one group who had secure counting skills and they could do the simple take away and adding and then you had the low lower attainers who were… one of them couldn't get past 20 on counting and another one that couldn't say, if you gave them a number, she couldn't say one thousand four hundred and twenty eight, even if you told her to say it, so it was a lot of speech problems as well, needed a lot of work on taking away from a hundred, because they could recognize numbers but they couldn't identify numbers with how to say them, so it would be like, if you gave them 33, they'd probably say 13 because that's the number they know.

R: Have you got a particular good memory from that placement, that involves you teaching maths?

S: Yeah, one of the children, he was a twin and he was the less dominant twin, and he used to erm… it wasn't a tantrum, but he'd sort of go into his shell and wouldn't even, the amount of talking you could do with him, he wouldn't… he'd just recoil and sit there and that's it, I'm not doing anything, and we were doing adding, no, we were doubling and halving numbers and I showed him the method where you break the hundreds and then you break the tens, because we didn't have any units, just the hundreds or tens, and he sat there and I sat with him for about ten minutes just going through it with him on a little whiteboard, and something just clicked, like that, and his enthusiasm for mathematics just went from being about 1 to about an 8 or 9 and he'd come in every morning and say to me 'Are we doing maths today?' and I'd say 'yes' and he'd be like smiling and beaming and he'd go and sit down, so for me that was personally because he, when I first went into the class, his twin was in the same group as him, so it wasn't a difference academically, but he just, because he was the less dominant twin, he was just like, you know?

R: That's fantastic! Well, that was the warm-up question, so now let's think about planning. How do you go about planning a mathematics lesson?

S: Erm... that's quite a hard question… when I was in school, I was given plans from the Hamilton Trust and I adapted those into my own planning to suit the class, but if I wasn't given those then I would probably use the Primary Framework, probably the 'I can' statement or something like that and the suggested tasks they have, because all you need is something little to get your brain going, so I'd probably use that and then go on.

R: So how would use it? What would you do with it?

S: Well I'd read it first, see whatever subject, what part of maths I'm teaching and I'd read that, then I'd probably think right, well I can do this, and probably do like a cloudburst with different ideas, obviously I'd have to know what the ability of the children are, and that's what would probably influence me… if I was doing supply teaching, I'd probably have lots of different little activities planned because I wouldn't know what the children can do.

R: So when you plan your lessons, which resources do you use to help you?

S: Yes, I used the Primary Resources website, that's really good for things like mental and oral starters, powerpoints, just for the visual learners, as well as the erm… the kinaesthetic learners as well because they obviously can touch the interactive board, and I used quite a lot of those in my planning, they would have sparked a lot of ideas as well of what I can do in the lesson to follow on from those, and worksheet…no, I know that sounds really bad, worksheets, but erm… worksheets that I could adapt that were fun, they weren't just two add two, it was things you could cut out and stick that were quite interactive.

R: What about other resources, books perhaps?

S: Erm… I don't really use books, I use the internet I think because people add and update, especially the Primary Resource site, they update that probably once a week, twice a week, so there's new ideas on there all the time.

R: Have you used the interactive planning tool on the online framework?

S: No.

R: Have you heard of it?

S: No!

R: Did the school have its own scheme for maths?

S: Not that I was aware of, I think what they were doing is they had a maths coordinator for Key Stage 2 and a maths coordinator for Key Stage 1, and because both of the teachers were part-time, so they took a Key Stage each, erm… and basically they were erm... a lot of them used the Hamilton Trust website, I think because it was a big… it was easy for them just to go print and it comes off, but I must say if you are using the Hamilton Trust work, it's really helped me understand progression in my training because its quite hard to see where something is going to go and you might pitch it too high or too low, but with that I could see, sort of, the main, the average Year 4 work and then pitch it to the different groups.

R: So would you say the Hamilton Trust resources really influenced the way you planned?

S: Yes, definitely.

R: Let's think about the examples you use in maths lessons. Do you think the choice of
examples you make is important?

S: Definitely, because I think the… especially with the class that I was working with, the children, they needed to be shown how to do something, and if you didn't show, if you showed… didn't show them how to do everything individually, every different aspect of erm… say, time, if you didn't show them how to do, how to show five past, ten past, and just explained to them that it was… that the time went down in fives round the clock, they wouldn't… you'd need to show them and explain to them explicitly using examples and I think especially in my planning, I've used a lot of examples at the beginning of the lesson and towards the end of the week I've faded them out slightly.

R: So what do you replace them with?

S: Well because they've understood the concept, especially with the higher attainers, erm... they, basically they understood the concept so that was locked in their brain so... I'd use more examples with the young... with the lower attaining children, but I think the higher attainers would understand the concept so therefore wouldn't need examples, they'd just need an explanation.
R: When you look at the resources such as Hamilton Trust or Primary Resources and you see the examples they provide, how much consideration do you give to those examples? Dp you look through them and decide whether they are all appropriate, or do you assume because they are on there, they must be alright? What do you do with the examples they present there?

S: Erm... it depends on time... it's a big influence, I think, because at the beginning of my placement I would sit there and read through them all and go through to see which ones I would use or which ones I would change slightly, whereas towards the end I was still looking at them but not as much in the sense that I would probably just read over them very quickly to make sure there wasn't any of them that were inappropriate, I wouldn't change them if they weren't.

R: Are there times when you have changed them?

S: Yes.

R: So what sort of things would make you change the examples?

S: Particularly for the lower attainers in the class, giving them big numbers, because a lot of the work I did in school was on numbers, only in the last couple weeks I did time, I think introducing the children to er... the lower attainers to 4-digit numbers, they were struggling with that so I didn't progress them onto the 4-digit numbers because they still needed work on the 3-digit numbers, so I would change the questions or the examples to 3-digit numbers just to suit them whereas the higher attainers and the middle group I would give the 4-digit numbers to, so there'd be a split in the examples.

R: So if you've got some 4-digit numbers on the website or on the sheets to give to the ones who can manage 4-digit numbers, would you still just give the numbers from the website or might you look at the actual four digits they have used and change that for any reason?

S: I did change some for the higher attainers in the class because a lot of the... when we were doing doubling and halving, for example, we had... we were just looking at the hundreds and tens, whereas... sorry, the thousands, hundreds and tens, but with the higher attainers I then changed it to the units as well, they had to split the units up as well, and I think a lot of the resources on the website only focused on the thousands, hundreds and tens so I changed those to make it more difficult and challenge for them.

R: After your starter, when you are going to do the main teaching input to your lesson and you are going to use some examples to demonstrate something, how do you decide which examples to use?

S: I think it's to do with how interactive they are, to start with, because, for example again, the class that I worked in, I had about six children in the class that were dyslexic, so they needed a lot of kinaesthetic activities where could actually stand up and go and touch things and move things around and I think for me, even though the examples, say four hundred and something, that probably wouldn't really play a part in my lesson so much as the interactivity of the examples that I was giving.

R: So where would you have got your examples from? Would you have taken them from Hamilton Trust or Primary Resources?

S: Some of them I took from Hamilton, and then some of them I was adding my own that were slightly different, but along the same lines as the Hamilton Trust, just to secure their understanding before sending them off on their own to do the work.

R: Were you thinking more about how you were going to use the examples?

S: Yes, rather then what the examples are, yeah.

R: Did you design your own worksheets or activity sheets for the children to use?

S: I did in a couple of lessons, erm… I didn't use many worksheets, a lot of the time I used either books, that were already there, just to save time, but also erm.. some of my tasks were quite interactive in the sense that they'd have it on the board and they could pick and choose what they wanted to do within reason, obviously!

R: So if you have a set of examples on the board, how do you know they are not going to pick out one particular type of example?

S: Well, what I had was I colour-coded them, some of them, so the harder questions were in red, some of the questions were in blue and then the easier ones were in green, but I asked them to choose sometimes which ones… they had to do all of one and then if they finished that then they pick one… they could either pick a blue or a red or…

R: When you sort the examples into colour groups, is it purely on level of ability?

S: Yes.

R: Nothing to do with the actual concepts, justeasy ones and harder ones?

S: Yes, because for example, we were using… we were 'timesing' numbers, 2-digit numbers by say 2 and 5, and they were the ones for the lower attainers in the class because they struggled with their… any times tables that weren't 1,2 or 5, so I'd put them into that colour, whereas the higher attainers could do all of their times tables so they were given 8s, 9s, 7s, and they were 'timesing'… because they had a really secure knowledge of their times tables.

R: Just going back to worksheets, if you did do a worksheet, and had a set of examples on a worksheet, how do you decide the sequence of examples on the worksheet?
S: I was usually putting an example at the top of the worksheet and then the questions underneath.

R: What was it that helped you decide what that example should be?

S: It would be quite random, in a way it's just the first question that comes into my head, or it's something we've covered in the lesson that I've put in at the beginning of the lesson, but then I've decided to maybe change one digit or something, just so they recognize the concept.

R: You said you used the books for the children sometimes, so if there's a page of examples in the book, do you look at those to decide which questions to use?

S: Yes, I did that quite a lot actually, in some of my planning I'd put maybe question 1 to 12 and then maybe 14 to 18 or something, so they miss out a few that maybe were either too easy for them or a harder concept they didn't understand.

R: So again, it's on ability?

S: Yeah.

R: Do you ever ask the children to make up their own examples?

S: Yes.

R: What sort of situation might you do it in?

S: We were doing problem solving and, especially with the higher attainers, if they finished their work, which they usually did very quickly, like they could finish a whole page of work in ten minutes if they understood the concept, I was getting them to think up questions of their own, I mean their own problem question, so we were looking at doing sorting lists to work out how many different possibilities of names there could be for children, or something like that, erm... and we sorted that out and then I asked them to think of their own question they could pass to their partner and then they had to then explain to the class, give their example to the class, of the question they've made up.

R: Right, now thinking about the course you've done, covering the theory about teaching maths, are there theories or models or some literature you've read that have helped you when you plan maths?

S: Probably the lectures for ideas, and the way in which to set out the lesson plans...erm... I think in particular there was one lecture where we weren't told what to do. She literally just gave us a piece of work or just some cards and just stood there, and I'm pretty sure this was about the second year or something, and erm... and they, she then reiterated the point that you need to tell them what to do, you need to show them what to do, and it was only for about two minutes, it wasn't a whole lecture, and that stuck in my brain, it stuck for me so whenever I plan a lesson, that's ticking away in the back of my head, you need to tell them how to do it before you give them the work, so that's what I try and do in all my planning.

R: So that's a practical teaching tip...

S: Mmm, yeah.

R: Have you used ELPS at all to help you plan maths?

S: Remind me what ELPS is...

R: Pamela Liebeck's model – experience, language, pictures, symbols.

S: No, I haven't. I know what it is, yeah, I just forgot what it stood for. No, I've never used it, I suppose subconsciously I probably use it in some of my lessons, but I've never consciously thought I'm going to use ELPS for this because I think it's for me, personally, it's dependent on the children as well or as much as the theory behind it.

R: So your only use of that theory has been in essays?

S: Yes, explicitly, yes.

R: What about VAK?

S: Oh yes, that's always in my planning, I always make sure I've got something visual for the children because I think, especially in the class that I was working in, we had about 90% of them were visual learners... erm... obviously you are going to talk through things and I always like to make lessons as interactive as possible and erm... and using programs on the interactive whiteboard or even just giving children sticky notes to position themselves when they are ordering numbers, just things like that and then I think I try and incorporate the whole of that into my learning and experience for the children... you can't always do it for everything but I try to at least incorporate at least two of those if not all of them.

R: Have you heard of a theory called the Knowledge Quartet?

S: I haven't, no.

R: Ok, just looking at a couple of your lesson plans, for example, a lesson here on 3-digit addition, you've got 437 plus 325, was that a random selection? Why is it those numbers in particular?

S: I think that was a random selection, as you can see it's from lesson 3 so it's the third one in the week, and it was getting the children to think about adding units that go over 10 so they are going to have to carry numbers, so, but I didn't want the whole number to go over 1000, so that was the thinking behind that example.

R: There's another one here, session 4, 482 plus 335, so what is the thinking behind that one, is it similar?

S: That's the same, yeah. It was erm... because the children needed a couple of lessons to understand how to carry over, it was the same sort of idea that I wanted it to go... it was the units didn't carry over but the tens did, so that's what I wanted to go onto, but it's still the carrying process.

R: On the next lesson on from that, you've got a whole set of examples, a lot of which seem to have the number 424 in. Why does that number come up so often?

S: That was actually taken from the Hamilton Trust, I don't know the reason behind that, but basically with this one I did take it from the Hamilton Trust because I think, the thinking behind that was that they've used 24 in practically all of their numbers but I don't know the thinking behind that, but it worked really well.

R: Now there's a subtraction one here, you've got 1000 subtract various numbers, and the first question you've got 1000 subtract 650, and the next one is 250, and then 150 and then 50. I wondered what was the reasoning behind the order of those?

S: I think erm... this was taken from the Hamilton Trust but I ordered them because erm... well if you look at the most recent one that they've done, [950 + 50] so because this is a harder question to answer, trying to link the two, I thought that if I did them together, then they are still going to understand how when you add something to 650 to get 1000, it's going to be the same as taking away 650 from 1000. So it was making a connection between the two, but with the class, they found it really hard to think of something and then go on to the next idea, there needed to be some sort of link between them.

R: Alright, the next one we've got the first session of the week, and on two ends of the counting stick we have 3246 and 3346. What is the significance of those numbers, other than they are 100 apart?

S: I think it was for me to give them a number that was erm... a long number and had, it didn't just have zeros in it, so it was a random selection but obviously there was still the guideline of it needs to have a digit erm... that doesn't have a zero in it in each place value, it had a number.

R: And was it deliberate to make it 100 apart?

S: Yeah, that was deliberate to make it 100 apart.

R: Right, second session for that week, you've got some prices, were they random?

S: They were random but we did relate them to what they could be, so the things were like a DS game, a CD, a DVD we related them to, but I got the children to relate them so it would make their learning more relevant, if I'd chosen something like a bottle of wine they probably wouldn't have understood that.

R: Right then, doubling and halving, you've got children having to halve some numbers, so you have 340, 460, 860, 340, 780, 540… when I looked at those I wondered why those numbers. Are they random? When you halve that one, you get an odd multiple of 10, in fact when you halve all of them you get an odd multiple of 10, and I don't know if that's a coincidence, or…

S: No…that wasn't coincidence! It just happened and I hadn't spotted it! They were random numbers that I picked and I didn't think of that!

R: There's one here about chunking, and some examples of how to chunk, and I wondered why those examples are there?

S: I think some of them are from Hamilton, and then I put in some easier ones like the 75 divided by 5 and the 90 divided by 5. I think I put in the dividing by 5s and then the dividing by 4s and 3s for the higher attainers, but I needed to put something, I needed to introduce something that was slightly easier and build up on the… onto that concept, so it was a bit of both.

R: I'm going to look at the worksheets at the back…is this one that you would have made up?

S: Yes, and I know there is a mistake in one of these!

R: How do you decide on the sequence?

S: This one was for the… from what I can remember, was for the higher ability group, and they were pretty random but towards the end I was running out of ideas so I was trying to think of harder ones for them so in a way, the first maybe three or four are easier in the sense that they are fully thought about in relation to the 12 times table and then I've tried to make them slightly harder.

R: And here, some differences between 3-digit numbers, would those be random?

S: Erm… random again, but I think consciously I made them harder as they go down, just for the fact that the top few have got a lot of zeros in, whereas as you get further down the whole of the number has got numbers.

R: So generally it gets harder as you go down?

S: Yes, I think so, but there are some pretty hard ones in there as well. What we were doing with this was we were trying to get to 100, so from 55 to 100, and then working out the difference from 100 to 250. The next one is easier because it's just an exact multiple of 100.

R: The next one is rounding up and rounding down… tell me about this one.

S: This one was random again, and was for the lower ability group and they really struggled with this, this was one of the first lessons, one of the first weeks that I was teaching full time so I was still getting used to them and they really struggled with this. So I think the next… one of the next lessons I really simplified it for them because they just didn't understand.

R: We talked about the theories and models, ELPS and VAK… was there anything in any of the lectures or anything you've read about how to choose good examples?

S: I don't recall anything but erm… I think throughout the degree you learn what a good example is and what a poor example is, and I think you… it's subconsciously… consciously you probably wouldn't think something is a really good example because the lecturer says it is, I think it's kind of drummed into you from the start about how to pick a good example. I don't think you consciously relate it back to lectures but I think from the start you are kind of given some guidance of where to go.

R: Do you work on the basis that because they are telling you examples, you assume they are giving you good examples?

S: Yeah!

R: They wouldn't give you bad ones?

S: No, I'd hope not!

R: If you knew there was a book or a journal or a website that said how to choose good examples for your maths lessons, would you consider using it?

S: I'd probably browse it but I wouldn't sit there for ages reading it… I think a lot of the time it's down to the teacher's discretion about the abilities of the children, I think the government or whoever would release that website would say this is a really good example for your mainstream Year 4 or Year 5 class, but they don't know… I don't think they'd take into account the range of abilities that you can get in a class.

R: Let's talk about subject knowledge. Do you feel confident about your mathematical subject knowledge?

S: To a certain degree, yes.

R: Enough to teach in primary?

S: Yes!

R: So if you had a bright Year 6, you feel you could cope with that?

S: Not initially, I don't think, but I think with a little bit of maybe... drawing on, sort of my knowledge, and using probably websites or something like that for a Year 7 or Year 8 class depending on how gifted and talented he was or she was, then I think I would be... I'd feel confident, but I think initially I'd be like 'aagghh! But then I know deep down I probably have got the ability to teach that, I just need to do a little bit of reading or just recapping my own knowledge.

R: So your general confidence for teaching primary – where do you think that has come from?

S: Because I did ok in my GCSEs? And coming here, this is going to sound really harsh on my peers but I don't feel like I struggled as much with mathematics as some people do so... I'm about middle I think.

R: So in GCSE what grade did you get?

S: I got a C.

R: And you think that's quite sufficient for you to teach primary?

S: Mmm Hmmm. I hope so!

R: If you needed to improve something, because you had a bright Year 6 or whatever,how would you go about doing it?

S: Erm... I would probably go and get myself one of those GCSE Letts books, and I quite like the GCSE BBC Bitesize website, I'd probably also go on the Primary Framework because they do Key Stage 3, maybe even Key Stage 4 depending on how bright he was, just looking at that and where I need to take the child. I think for me it wouldn't be so much the knowledge because I think you can... as a teacher you can have sort of, a piece of paper with it on, just in case you're not 100% sure, just there to help you, but I think for me it's more the progression of the child and where to take them, because we are only given Key Stage 1 and Key Stage 2 and we probably have been given a little bit of Key Stage 3, but for me it would be where to take them through the year.

R: When we talk about mathematical knowledge, what do you think that means? What does it include?

S: I think it's the raw maths really, I don't think it's how to teach, it's knowing the facts and the concepts and the skills that you can then transfer.

R: So it's the pure knowledge, not how to teach it?

S: Yes, because I think anyone could learn maths but I think teaching is a different skill altogether.

R: And of those two types of knowledge, the maths itself and how to teach it, do you think there has been one or the other pushed on the course, or a bit of both?

S: I think it's a bit of both, I think in the first year it was more knowledge based, erm… the second and third year a lot of it has been giving you ideas of how to teach concepts and skills and things, and mental and oral starters, things like that.

R: Do you think your level of maths knowledge, whichever type you go by, helps you choose good examples?

S: In some areas of maths, I think, yes, because you are always going to have a memory that sticks out, either an example or how one of your secondary school teachers taught you something, and then you can obviously apply that to your lesson, but sometimes I think it's just good teaching to choose a good example to suit your class, I think you've got to know your class to choose a good example.

R: So somebody who may have done grade A GCSE and A-level maths, and is teaching Year 2, are they going to be better placed to choose examples than someone who got a C?

S: No. I don't think so at all, no. I think it all comes down to knowing your class to choosing effective examples, I think someone who got an A at GCSE or an A-level is going to baffle the children because they are only Year 2, their concept of… their maths skills are very limited.

R: GCSE and A-level tends to test the maths knowledge, pure maths, not how to teach maths, so does…

S: I wouldn't dispute the fact that they've got good maths knowledge.

R: Do you think people need better knowledge of how to teach maths to choose good examples?

S: Yes. Only primary, I think secondary school would be different.

R: Ok, so if you wanted to choose better examples, you're saying it's better to have good knowledge about how to teach maths.

S: Mmm.

R: Has that knowledge been given to you on this course?

S: Yes.

R: Sufficiently?

S: Yes, definitely. I think especially the lectures where… 'cause for me personally, I'm a visual learner and I think given when the lecturer comes in with her little truck of resources it really helps me visualize how I can deliver a lesson, and that obviously incorporates giving examples.

R: What do you think you've learnt about how to plan maths from seeing experienced teachers teaching maths?

S: Not a lot.

R: Not a lot? There's nothing you can think of?

S: No. Because everything that erm… in all the schools that I've worked in, you know, had my placement in, I've been handed QCA schemes of work, well, not QCA schemes, you know, the schemes of work, and I mean, in the final placement I was handed the Hamilton Trust and the Hamilton Trust is amazing, but I don't think I... if someone had given that to me, it's a lot of writing, so I think a lot of the time you need to sit there and go through it. There are teaching skills that I probably learnt from the teachers but not how to plan.

R: Do you think you would have benefited more from having more sessions on this course about how to teach maths to choose good examples?

S: No, because I think we've had quite a lot, even though we haven't been able to put our... put the things we've learnt into practice, I think we've seen so many good examples of how to teach and how to teach effectively that we can... it's kind of locked in now. I mean, I've got friends that went to a different university and the way... when we've talked about teaching and things I think 'gosh that's really bad' and I know that sounds really harsh or maybe that's the way I've been taught here, but erm...

R: So you think you had a better deal here?

S: Yeah, definitely because… I don't know, maybe it's just the way because it's slightly different here to another university, maybe they are better at something else, but I think maths here has really… it's so diverse, the learning, that it gives you an example of everything, from planning to mathematical knowledge to teaching to giving good mental and oral starters and things, and it's the range I think whereas they may get channeled down to the mathematical knowledge route, I don't know…

R: Which university was that?

S: (*deleted for confidentiality*). But they do a lot of maths, they can choose double maths and I don't know whether that's where they fall down, because they can choose maths as a strength which I think is quite a let down.

R: Thank you.

## Appendix 9

### Letter to trainees re: interview transcripts

Dear

I hope you are well and enjoying life after University!

Please find enclosed, as promised last term, a copy of the transcript from your interview with me towards my Education Doctorate which you may keep.

I would be grateful if you would read it through and then **if you feel there are any inaccuracies or errors, please let me know by email.**

Given that it took place about 3 months ago, I don't suppose you will remember every detail from the interview, and you may even be surprised when you read some of the things you said! Many interviewees are often amazed how often they use utterances like 'Erm' and 'Well, like…' and so on, but for accuracy I have left all of these in the transcripts.

I will use selected responses from the full set of interviews when writing up my thesis and any research and conference papers that come from the study, but I would like to remind you that any reference to you, your responses or your lesson plans will always be anonymous by use of a pseudonym.

If you are interested in seeing a copy of the final thesis when it is complete, please email me so I can contact you when it is published.

I am also happy to discuss any aspects of the interview with you if you feel it would help your professional development.

Many thanks again for your participation.

Best wishes

Ray Huntley
Centre for Academic Writing and Numeracy Skills
University of Gloucestershire
rhuntley@xxxx.xx.xx

## Appendix 10: Sample Lesson Plans

**ED18 Sharon**

| **Date** 29/1/08 | **Time** 11.15-12.10 | **Subject** Numeracy |
|---|---|---|
| **Development matters refs / NC links and reference to PNS for this lesson** *(must be written in full before mid-point review, after which, letters and numbers only are acceptable)*<br>NC KS2 Maths Ma2 Number<br>3e) subtract any pair of two digit whole numbers<br>3i) use written methods to subtract positive integers less than 1000<br>PNS Maths Y4 Calculating-<br>• Refine and use efficient written methods to subtract two-digit and three-digit whole numbers<br>• Add or subtract mentally pairs of two-digit whole numbers | | |
| **Difficulties, errors and misconceptions that children may have in this lesson**<br>Some children may find it difficult to partition numbers. | | |
| **Learning objectives for the lesson**<br>*What do you want the children to learn?* (To be able to ....)<br><br>TBAT use a hundred square to subtract tens and units. | **Assessment Questions**<br>*How are you going to assess the learning objective? How will you know that the pupils have achieved the LO ?*<br>Can the children correctly answer the subtraction questions using the hundred square to help them?<br><br>Can the children correctly partition numbers? | **Target Assessment Pupils**<br>*Who will you focus upon & when will you assess?*<br><br>All children through questioning and observation<br><br>Dissertation children to see if they are using the placemats |
| **Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs**<br>Give placemats to J, C, N, G, W, M | | |
| **New vocabulary**<br>Less than | | |
| **Management of other adults**<br>Introduction<br><br>Briefing sheet: Yes/No<br>Yes/No<br>Name of adult:<br>Mrs C<br>Mr G | Main Part<br>Plenary<br>Briefing sheet: Yes/No<br><br>Name of adult: | Briefing sheet:<br><br>Name of adult: |
| **Pre-lesson preparation:**<br>**personal organisation and resources** | | |

| |
|---|
| Number grid<br>Key vocabulary for subtraction<br>Hundred grid<br>IWB<br>Small hundred squares<br>Placemats |

| | **STRATEGIES Please use headings as guidelines** |
|---|---|
| **Time** | **Introduction**<br><br>*For students working within Foundation Stage settings, please make it clear when this part of the session will be taught as it may be on-going throughout the week*<br>• Practice walking backwards again after yesterday's lesson<br>• (eg. 9+1=10, work out the subtraction sum).<br>• Invite children to come up and write answers on the IWB.<br>• Use more difficult sums so the children can see the theory<br>• Eg. 99-20=79 |
| | **Development (include Differentiated Activities)**<br>*For students working within Foundation Stage settings, please make it clear when this part of the session will be taught as it may be on-going throughout the week*<br>• **Key word: less than**<br>• Encourage children to use the key word when they are answering questions<br>• Put a 100 square on the IWB<br>• Show how to move across the number grid to take away tens and units.<br>• Ask children some questions<br>• eg. 30-10<br>• eg. 29-5<br>• Children to write the answers to given questions on their whiteboards, using the 100 grid on the board<br>• Ask children to explain their answers<br>• Give each child a number grid that they can use at their desk<br>• Give them some partitioning questions<br>• Eg. 50-25, so they must partition it first as 20 and 5, then subtract this using the method on the number grid<br>• Write some questions on the board for the children to answer in their books including the question and answer<br>Extension<br>• Ask children to try some harder partitioning questions with some help<br>• They must include all steps in their working ie. Partitioning, then the subtraction of the tens, then subtraction of the units |

| | |
|---|---|
| | **Plenary**<br>(include details of homework activities related to this lesson)<br>*For students working within Foundation Stage settings, please make it clear when this part of the session occur as the main part may have been an on-going activity throughout the week*<br><br>• Ask children to try to mentally subtract tens or units<br>• Eg. mentally subtract 32-10<br>• 31-20<br>• Move onto some that include partitioning too if they are understanding.<br>• Eg. mentally subtract 22-11 |

**Sample Lesson Plan**
**Suzy**

**ED06**

| **Date** 04.02.08 | **Time** 11:00 – 12:15 | **Subject** Numeracy (multiples) |
|---|---|---|

**Stepping Stone refs / NC links and reference to PNS for this lesson**

NC. Ma2 Number:
1b.'break down a more complex problem or calculation into simpler steps before attempting a solution; identify the information needed to carry out the tasks'
1f.'organise work and refine ways of recording'
1g.'use notation diagrams and symbols correctly within a given problem'
3a. 'develop further their understanding of the four number operations'
3f. ' recall multiplication facts to 10x10 and use them to derive quickly the corresponding division facts'
3h. ' multiply at first in the range of 1 to 100 then for particular cases of larger numbers…'
3j.' use written methods for short then long multiplication'
4a.'choose, use and combine any of the four number operations to solve word problems'

PNS Yr 5
'quickly recall multiplication facts up to 10x10
'refine and use efficient written methods to multiply HTU x U'

**Difficulties, errors and misconceptions that children may have in this lesson**
Using the methods correctly
Doing enough work – getting on

| **Learning objectives for the lesson** | **Assessment Questions** | **Target Assessment Pupils** |
|---|---|---|
| -to practise working out U x TU problems mentally<br>- Use grid and column method for working out HTU x U<br><br>TA grp:<br>-Use grid and column method for working out TU x U money problems<br>-introduce using grid and column method to work out HTU xU<br>**Ext:** Use grid and column method for working out TU x TU | • *Can I use the grid method to work out HTU x U questions?*<br><br>• TA grp: *Can I use the table and column method to work out TU x U questions?*<br><br>• EXT:*Can I use the grid method to work out TU x TU questions?* | • JW<br>• CO<br>• J<br>• N |

**Points for Action, related to teaching & learning from previous evaluations/ pupil assessments & pupils' IEPs**
Observe how they work in a different set-up (ability grouping)
Recap on mental multiplication – to help working out the problems
Divide group up to make differentiation easier

**Vocabulary** multiple multiply add

**Management of other adults**
TA will take the lower ability group out
CT and I will be present to help the other two groups

**Pre-lesson preparation: personal organisation and resources**

| | |
|---|---|
| books to record<br>the factors of?<br>worksheets<br>Calculators | IWB – squared screen and blank screen –what are<br><br>Individual whiteboards and pens<br>TA- H4 books and indiv. Whiteboards and pens |

| | STRATEGIES Please use headings as guidelines |
|---|---|
| **Time**<br><br>10min | Introduction<br><br>Sort out groupings – rearrange seating<br>1. Ta's group (lower)<br>2. sat at centre table and window table (middle)<br>3. sat along the sides and back (higher)<br>Do a few factors on the board – number in centre and factors round them. |
| 15min<br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br><br>40min | **Main**<br>**Teaching Part**<br>Working out TxU problems mentally – only 10s. e.g 6 x 40<br>What's 6 x 4? X10 (add a zero) etc<br>Do a few.<br>Individual whiteboards – do a few – whole class<br>Then look at H x U – e.g 3 x 400<br>What's 3 x 4? 12 then x 100? (add 2 zeros) 1200<br>Do 2 on individual whiteboards again.<br>Show them demo of using grid method and column method – leave to refer to.<br><br>**ACTIVITY:**<br>Do worksheet<br>If you finish the first part tell me before continuing.<br>Go over TUxTU on IWB<br>-ask children to come up and write in the numbers as appropriate<br>-discuss steps<br><br>**Ext:** TU x TU<br>**TA's grp:**<br>H4 pg 48 Q.3 solving TUx U money problems<br>Go on to look at HTU x U – on individual whiteboards |
| 10min | **Plenary**<br>TA's group now join: (sit at separate table)<br>Calculator game: Down from 100. Hand out calculators<br>I set a number which they have to put in.<br>They must make TWO moves – can be + x – or ÷<br>To try and get it down to the number I give them – should use two different ones if possible.<br>e.g. to 20 100÷4 = 25 25-5 = 20<br>**Differentiate numbers for each table** |

**Sample Lesson Plan**
**Rachael**

**ED26**

| Date 9th March, 2009 | Time 10.55am – 12.00pm | Subject Numeracy |
|---|---|---|

**Development matters refs / NC links and reference to PNS for this lesson** *(must be written in full before mid-point review, after which, letters and numbers only are acceptable)*

**2(a)** To count on and back in tens or hundreds from any two-digit or three-digit number.
**2(b)** To recognise and describe number patterns including two and three-digit multiples of 2, 5 or 10.

**Difficulties, errors and misconceptions that children may have in this lesson**

- Some children may be concerned about adding to 3- and 4-digit numbers.

| Learning objectives for the lesson | Assessment Questions | Target Assessment Pupils |
|---|---|---|
| **To recall multiples of 10 and 5.** | Can the children recall multiples of 10 and 5? | TGA (RS) Diamonds |
| **To add and subtract a multiple of 10 to and from a 3-digit number, crossing 100.** | Are the children able to add and subtract a multiple of 10 and from a 3-digit number, crossing 100? | TGA (SB) Circles |
| | | Traffic light outcome. |

**Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs**

- From previous lessons I am aware that children in the Diamonds group lose confidence when asked to add large numbers. For this reason I will work with these children to ensure that they are supported. (From numeracy lessons during week commencing 02.03.09)

**New vocabulary**

- Multiple, 3-digit number, 4-digit number, addition, subtraction.

**Management of other adults**

Introduction

Main Part

Plenary

Briefing sheet: Yes/No
sheet: Yes/No
Name of adult: N/A
adult: N/A

Briefing sheet: Yes/No

Name of adult: N/A

Briefing

Name of

**Pre-lesson preparation:**
**personal organisation and resources**

- Numbers laminated,
- Dice,
- Work books.

| | STRATEGIES Please use headings as guidelines |
|---|---|

| | |
|---|---|
| | |
| **Time**<br><br>**5-10 minutes** | Introduction<br><br>• Children will use the IWB game to practise recognising multiples of 10 and 5.<br>• See if the children can beat each other's scores.<br>• How can we tell if a number is a multiple of 10 or 5?<br>• Game is from the Maths Zone website. |
| **10 minutes**<br><br>**25-30 minutes** | **Development (include Differentiated Activities)**<br><br>• "Today we are going to continue to add multiples of 10 to 3- and 4-digit numbers".<br><br>**<u>Whole class:</u>**<br><br>• Begin by choosing one number to add and subtract 10 to and from, slowly explain the method.<br>• Then progress to 2 and 3-digit numbers, explaining the method that is used each time and how to set the work out.<br>• Explain that each group has a different task and inform children of these tasks.<br><br>**<u>Group work – differentiated by task:</u>**<br><br>The children will complete differentiated tasks.<br><br>**Triangles:** Provide the children with a selection of number cards. The children will choose four cards and write the number (e.g. choose 1456). The children will then add 10 to the first 5 numbers they choose. They will then subtract 5 from the next numbers they choose. (Write the multiples they are to add and subtract by on the whiteboard.)<br>**Squares:** Children are to take one card from a pot which will be a 3-digit number. The children will then take a card that is a multiple of 10. The first number card will read 345 + or 345 – to inform children of sum.<br>**Diamonds:** Children will roll a dice to get a two digit number and will add multiple numbers to this number. Work with the children.<br>**Circles:** Children will work with S.B. and will use cubes to help with adding a 2-digit number to 10. Children will begin to look at subtracting. |
| **5-10 minutes** | **Plenary**<br><br>• Choose 3 children to say a number.<br>• Write the number sentence.<br>• Ask the children to find the answer and explain the method they used. |

**Sample Lesson Plan** **ED24 Naomi**

| Date 04/03/09 | Time 11am-12pm | Subject Maths |
|---|---|---|
| **Development matters refs / NC links and reference to PNS for this lesson** | | |
| **Difficulties, errors and misconceptions that children may have in this lesson**<br>• **Knowledge of number.** | | |
| **Learning objectives for the lesson**<br><br>**To be able to use knowledge of number to work out mathematical problems.**<br><br>**To be able to explain methods and reasoning orally when solving the puzzles** | **Assessment Questions**<br><br>Can the child use their knowledge of number to work out problems?<br><br>Can the child show a method when trying to solve the mathematical puzzle?<br><br>Can the child give reasons for their methods and explain this orally? | **Target Assessment Pupils**<br><br>Squares- (see class assessment 3/3/09) some children need help to solve mathematical problems with the guidelines given. |
| **Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs**<br>Write learning objectives on IWB and share these with the class.<br>Differentiation.<br>Time management<br>Explain clearly | | |
| **New vocabulary**<br>Multiples, subtraction, negative numbers. | | |
| **Management of other adults**<br>Introduction<br><br>SH to work with H and E<br>AS to work with EG and EM<br><br>Main Part<br>Plenary<br>SH to work with Triangles<br>AS to work with EG and EM<br><br>N/A<br><br>Comments- SH to focus on H and E working through an understanding of number bonds | | |

| **Pre-lesson preparation:** **personal organisation and resources** |
| --- |
| Whiteboards, pens, rubbers, IWB, rulers and Books.<br>Make sure books are placed in the middle of the table with rulers |

| Time | <u>Introduction</u> |
| --- | --- |
| 15 mins | **M&O starter-**<br>Wizard zap- The children are put into teams and asked to answer a Q the child that answers first with the correct answer zaps someone in their team. Last one standing wins.<br>**SH- to work with H and E out of the class room**<br>Write on the IWB 42+25, 42+52, 64-15, and 64-52.<br>You can work these out in your head but we are going to use a calculator. However, the 5 button is broken.<br>**Q- How can we work this out?**<br>Repeat this for all of the sums.<br>Get the children to work this out on their whiteboards in pairs.<br>**Q- How did you work this out?** |
| 30 mins | <u>Main activity</u><br><br>**Share the WALT- WALT solve mathematical puzzles**<br><br><u>Triangles</u><br>Using digit cards make the square up on the table and ask the children to work out where the numbers go so each row and column adds to 15.<br>If they get stuck suggest putting the even numbers in the corners.<br>Ask them to explain their findings and if they notice any patterns.<br>Make sure they draw the finished square in their book.<br>Then try this with a number square that has 4 in the middle and use numbers 0-8.<br><u>Squares and Pentagons</u><br>Children come up with equivalent calculations for<br>82+35, 75 + 24, 87 + 56, 54 + 32, 87 – 25, 75 -21, 96 – 52, 56- 23.<br><u>Hexagons</u><br>Children come up with equivalent calculations for<br>82+35, 75 + 24, 87 + 56, 54 + 32, 87 – 25, 75 -21, 96 – 52, 56- 23.<br>Then 32x5, 15x2, 80/5, 105/5. |
| 10 mins | <u>Plenary</u><br><br>Reflect upon the activity and whether the children feel they have met the learning objectives of the lesson.<br>Place 20 x 5 ask someone to explain their findings and working out. |

**Sample Lesson Plan** **ED21**
**Victor**

| Date 12.3.08 | Time 11:00am | Subject Numeracy |
|---|---|---|
| **Development matters refs / NC links and reference to PNS for this lesson**<br>**PNS: Knowing and using number facts.**<br>**NC KS2: Ma2: 2a,b** | | |
| **Difficulties, errors and misconceptions that children may have in this lesson**<br>• Most children are struggling to use their knowledge of number facts to solve a calculation. They are phased by larger numbers, particularly 3 digit numbers. | | |
| **Learning objectives for the lesson** | **Assessment Questions** | **Target Assessment Pupils** |
| • Children are to use the column method to reinforce adding and subtracting multiples of 10 to and from 2 and 3 digit numbers. | • *Can a child use the column method effectively to help them to learn how to add and subtract multiples of 10 to and from 2 and 3 digit numbers?* | • *WILP*<br>• *WILC*<br>• *CHA*<br>• *JOR* |
| **Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs**<br>• The TAP children were very poor yesterday and failed to meet the LO. Hopefully with the introduction of the column method the children will be able to do more working out on paper that will hopefully help them to meet the requirements outlined in the relevant statutory frameworks. | | |
| **New vocabulary**<br>Column, method, calculate | | |
| **Management of other adults**<br>Introduction<br>Briefing sheet: Yes/No<br>Name of adult:<br>of adult:<br><br>Comments<br><br>I will support the TAP children and the CT will support JOS and DOT. | Main Part<br>Briefing sheet: Yes/No<br>Name of adult: | Plenary<br>Briefing sheet: Yes/No<br>Name |

| | STRATEGIES Please use headings as guidelines |
|---|---|
| 15 | **Introduction: Starter: Locate the position of a 2 digit number on a number line.**<br>• Sit children down at the carpet.<br>• Explain to the children the LO<br>• Work through with the children examples of prior experience in calculations.<br>• Do **34 + 50 =, 245 + 40 =**<br>• Next discuss with the children the column method. Ask Q's and ensure that the children understand the modelling activity by participating.<br>• Do **43 + 30, 367 + 70, and 965 + 80** as a column addition. Ask the children to support me and give me answers.<br>• Guide the children through the lesson expectations and send them back to their seats. |
| 30 | **Development (include Differentiated Activities)**<br>• Provide the children with the differentiated sheets.<br>• Ask one of the children to hand out number squares and number cubes.<br>• Encourage the children to use the spaces on their sheets to work out in if they need to.<br>• Sheets 1-3 should be given to the relevant children as yesterday, 1 being the most challenging and 3 the least.<br>• If children are quick to finish they may have a go at stepping up a level.<br>• If they have been significantly challenged by their sheet they may play the dice game.<br>• They may work individually or in pairs.<br>• They must roll a multiple sided dice and add and subtract that number from a number card they have chosen from a random stack.<br>• They can record the sum in their books or on the work sheet and use whichever method they want to get the answer. |
| 15 | **Plenary:**<br>• Get children to pack up and then sit on the carpet in pairs with a whiteboard and pen.<br>• Consolidate today's learning and assess the children by discussing their work and analysing their feedback.<br>• Write the number 77 on the IWB.<br>• Ask the children to write down a couple of additions for which this number is the answer.<br>• Show and discuss. |

**Sample Lesson Plan** **ED13**
**Dawn**

| Date 28/1/07 | Time 9.30-10.30 | Subject Numeracy |
|---|---|---|
| **Stepping Stone refs / NC links and reference to PNS for this lesson**<br>NNS Yr 4 Use knowledge of addition and subtraction facts and place value to derive sums and differences of pairs of multiples of 10, 100 and 1000<br>NNS Yr 4 24 Relate fractions to division and find simple fractions of numbers or quantities | | |
| **Difficulties, errors and misconceptions that children may have in this lesson** | | |
| **Learning objectives for the lesson**<br>*What do you want the children to learn?* (To be able to ....)<br>• Make number pairs to 100, 1000 (multiples of 50), 1 (decimals)<br>• Find simple fractions of quantities of objects | **Assessment Questions**<br>*How are you going to assess the learning objective? How will you know that the pupils have achieved the LO ?*<br>• Observation and questioning<br>• Assessment of written work | **Target Assessment Pupils**<br>*Who will you focus upon & when will you assess?*<br>• Owen, Jessica and Joe by observation during the activity and assessment of written work |
| **Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs** | | |
| **New vocabulary**<br>Fractions, halves, thirds, quarters | | |

| | STRATEGIES Please use headings as guidelines |
|---|---|
| **Time** | **Introduction**<br><br>Remind children about work recently on number bonds to 100, 1000 and 1.<br><br>Give each child a card, initially face down.<br><br>Ask everyone with a yellow card to stand up, look at card. They have 30 seconds to find partner to make bond to 100. Ask pairs to stand together at front.<br><br>At end of time, ask other pupils if the pairs are correct. Ask them to match up any remaining pupils who haven't found partner.<br><br>Congratulate pupils! Ask them to sit down. Repeat with Blue cards (pairs to 1000), then Green cards (pairs to 1) |
| | **Development (include Differentiated Activities)**<br><br>Show children collection of 12 marbles in a pot. Ask a child to take out half of them. How many were taken?<br>Ask other children if that is right. Ask child how they worked out half of 12. Any other suggestions? (Looking for 'divide by 2')<br><br>Ask another child to take out a third of the remainder. Discuss as above.<br><br>Ask another child to take out one quarter of the remaining marbles. Discuss as above.<br><br>Children to work through examples in Abacus Textbook 3, Yr 4, p73. Write in books. Can move on to p74. |
| | **Plenary**<br>**(include and details of homework activities related to this lesson)**<br><br>Target questions at children, eg if I had 12 cats, what would be one third of them? Child who answers correctly can ask next question. |

**Sample Lesson Plan**
**Andy**

**ED37**

| Date | 12/03/09 | Time | 10.55 – 12.00 | Subject | Mathematics |
|---|---|---|---|---|---|

**Development matters refs / NC links and reference to PNS for this lesson**

1d: find different ways of approaching a problem in order to overcome any difficulties; 3a: develop further their understanding of the four number operations; 3d: recall all addition and subtraction facts; 4b: choose and use appropriate way to calculate and explain their methods and reasoning.

**Difficulties, errors and misconceptions that children may have in this lesson**

Children may have difficulty adding and subtracting decimals, children may have difficulty understanding tenths and hundredths may have difficulty using methods correctly to add and subtract decimals.

| Learning objectives for the lesson | Assessment Questions | Target Assessment Pupils |
|---|---|---|
| • *Mentally add or subtract a pair of decimal numbers, crossing units of tenths.*<br>• *Use known number facts and place value for mental addition and subtraction of decimals.*<br>• *Find what to add to a decimal to make the next* | Can children add and subtract decimals?<br><br>Do they use methods correctly to add and subtract decimals? | Less Able Group<br><br>Green and Purple Tables<br><br>A /S |

**Points for Action, related to teaching and learning from previous evaluations/ pupil assessments and pupils' IEPs**

Ensure that all children remain on task, explain that they will have to complete work in break times, ensure all children sit straight, still and listen on the carpet, encourage children to put hand up and stay in seat if they want help.

**New vocabulary**

Addition, strategies, *partition, tens, units, tenths, subtraction, column addition/subtraction, metal maths, one whole number, split, hundredths, method, decimal.*

**Management of other adults**

| Introduction | Main Part | Plenary |
|---|---|---|
| Briefing sheet: Yes/No<br>Name of adult: | Briefing sheet: Yes/No<br>Name of adult: | Briefing sheet: Yes/No<br>Name of adult: |

Comments

- Mr M / to work with lower ability group;
- Miss A' ... to work with middle group (yellow) ensure A' : remains on task;
- Mrs M y to work with A

| | **STRATEGIES** Questions/Key Vocabulary/**Differentiation** |
|---|---|
| **Time** | **Introduction**<br><br>**Mental Oral Starter: Mr M ’s Price is Right. (Higher or Lower.)**<br><br>Two children to stand at front of the board facing the class. Children turn cards (5x3) answer sum and gain a number. Children guess if next card (higher or lower) if correct continue, if incorrect swap player. All children work sum out to check player's answer on mini whiteboards. |
| | <u>**Development (include Differentiated Activities)**</u><br><br>Write on board: 4.5 + 1.8 =. Write both numbers on a place-value grid with units and tenths on it. Draw a number line marked in tenths. Use athletes race times as an example of using units and tenths.<br><br>*Demonstrate strategy:* Ask child to mark 4.5 on number line. *How places do we have to jump to reach next whole number? What have we added?* (0.5) Demonstrate how to add or subtract numbers that do not make a whole unit: *3.5 = 3 + 5 tenths and 4.3 is 4=3 tenths. Add them together= 3+4=7, and 5tenths=3tenths= 8tenths. 7+8tenths = 7.8.*<br><br>Demonstrate adding decimal numbers: Write on the board: 3.8 + 1.6 = _. Pint at the number and remind children that we can split them into the units part, and the decimal or tenths part. Draw a place-value grid and write the numbers on it. Rewrite the addition, splitting the numbers.<br><br>Write on the board: 3.8 – 1.6=_. Explain that we can use the same method of splitting into units parts and tenths parts to solve this subtraction. Demonstrate: 3-1 = 2 and 0.8 – 0.6 = 0.2, so 1.8 – 1.6 = 2.2.<br><br>**Activity:**<br><br>**Less able:** To complete diffrentiated activity sheet: add and subtract simple decimal numbers: *4.6 + 5.7 = ?* and *6.3-4.3= ?*Drawing numberlines in books to help them. Encourage to do mentally.<br><br>**Middle ability:** To complete diffrenetiated activity sheet: to add and subtract decimal numbers, move onto more challenging decimals and a sequence of decimals: *4.6+5.3+2.3=* and mixed add/subract decimals sums: *5.4+4.3+6.5-6.4=?*<br><br>**More able:** To complete differentiated activity sheet: To complete challenging decimal addition/subtraction, word problems and larger sequences. |
| | <u>**Plenary**</u><br><br>Write on the IWB: 4.2 + 9.3 + 3.3 = ? Children to work out answer mentally and show how they did it. *What strategy did you use?* |

**Appendix 11a: GCSE grades for the 2005-08 cohort: the percentage of each grade for the cohort and the sample group**

| GCSE Grade | No. of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| A* | 1 | 1 | 1 | 5 |
| A | 18 | 16 | 4 | 18 |
| B | 33 | 30 | 7 | 32 |
| C | 59 | 52 | 10 | 45 |
| C Equivalence | 1 | 1 | 0 | 0 |
| **Totals** | **112** | **100** | **22** | **100** |

**Appendix 11b: GCSE grades for the 2006-09 cohort: the percentage of each grade for the cohort and the sample group**

| GCSE Grade | No. of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| A* | 3 | 4 | 0 | 0 |
| A | 9 | 13 | 2 | 13 |
| B | 30 | 43 | 6 | 38 |
| C | 27 | 39 | 8 | 50 |
| C Equivalence | 1 | 1 | 0 | 0 |
| **Totals** | **70** | **100** | **16** | **101*** |

* = rounding of percentages has resulted in a figure different to 100%

**Appendix 12a: GCE A level grades for the 2005-08 cohort: the percentage of each grade for the cohort and the sample group from available data**

| GCE Grade | No. of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| A | 2 | 14 | 1 | 33 |
| B | 4 | 29 | 1 | 33 |
| C | 3 | 21 | 1 | 33 |
| D | 3 | 21 | 0 | 0 |
| E | 2 | 14 | 0 | 0 |
| **Totals** | **14** | **100** | **3** | **99*** |

**Appendix 12b: GCE A level grades for the 2006-09 cohort: the percentage of each grade for the cohort and the sample group from available data**

| GCE Grade | No. of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| A | 0 | 0 | 0 | 0 |
| B | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 0 |
| E | 1 | 100 | 1 | 100 |
| **Totals** | **1** | **100** | **1** | **100** |

* = rounding of percentages has resulted in a figure different to 100%

**Appendix 13a: Interview test scores for 2005-08 cohort and proportions of cohort and sample group**

| Interview Score | Number of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| 0 or 5 | 0 | 0 | 0 | 0 |
| 10 or 15 | 2 | 2 | 1 | 5 |
| 20 or 25 | 4 | 4 | 0 | 0 |
| 30 or 35 | 14 | 13 | 4 | 18 |
| 40 or 45 | 14 | 13 | 4 | 18 |
| 50 or 55 | 28 | 27 | 4 | 18 |
| 60 or 65 | 19 | 18 | 3 | 14 |
| 70 or 75 | 11 | 10 | 1 | 5 |
| 80 or 85 | 7 | 7 | 2 | 9 |
| 90 or 95 | 4 | 4 | 1 | 5 |
| 100 | 2 | 2 | 2 | 9 |
| **Total** | **105** | **100** | **22** | **101** |

**Appendix 13b: Interview re-test scores for trainees in 2005-08 cohort who failed their first attempt**

| Re-test score | No. of students | % of students | No. of students in sample group | % of students in sample group of cohort |
|---|---|---|---|---|
| 0 or 5 | 0 | 0 | 0 | 0 |
| 10 or 15 | 0 | 0 | 0 | 0 |
| 20 or 25 | 0 | 0 | 0 | 0 |
| 30 or 35 | 0 | 0 | 0 | 0 |
| 40 or 45 | 0 | 0 | 0 | 0 |
| 50 or 55 | 15 | 60 | 3 | 50 |
| 60 or 65 | 8 | 32 | 3 | 50 |
| 70 or 75 | 1 | 4 | 0 | 0 |
| 80 or 85 | 0 | 0 | 0 | 0 |
| 90 or 95 | 1 | 4 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 |
| **Total** | **25** | **100** | **6** | **100** |

**Appendix 14a: Interview test scores for 2006-09 cohort and proportions of cohort and sample group**

| Interview Score | Number of students | % of students | No. of students in sample group | % of students in sample group |
|---|---|---|---|---|
| 0 – 10 | 1 | 1 | 0 | 0 |
| 11 – 20 | 0 | 0 | 0 | 0 |
| 21 – 30 | 1 | 1 | 0 | 0 |
| 31 – 40 | 4 | 6 | 2 | 13 |
| 41 – 50 | 5 | 7 | 1 | 6 |
| 51 – 60 | 10 | 15 | 1 | 6 |
| 61 – 70 | 15 | 22 | 1 | 6 |
| 71 – 80 | 18 | 26 | 7 | 44 |
| 81 – 90 | 12 | 18 | 4 | 25 |
| 91 – 100 | 2 | 3 | 0 | 0 |
| **Total** | **68** | **100** | **16** | **100** |

**Appendix 14b: Interview re-test scores for trainees in 2006-09 cohort who failed their first attempt**

| Re-test score | No. of students | % of students | No. of students in sample group | % of students in sample group of cohort |
|---|---|---|---|---|
| 0 – 10 | 0 | 0 | 0 | 0 |
| 11 – 20 | 0 | 0 | 0 | 0 |
| 21 – 30 | 0 | 0 | 0 | 0 |
| 31 – 40 | 0 | 0 | 0 | 0 |
| 41 – 50 | 0 | 0 | 0 | 0 |
| 51 – 60 | 2 | 18 | 1 | 33 |
| 61 – 70 | 7 | 64 | 2 | 67 |
| 71 – 80 | 0 | 0 | 0 | 0 |
| 81 – 90 | 2 | 18 | 0 | 0 |
| 91 – 100 | 0 | 0 | 0 | 0 |
| **Total** | **11** | **100** | **6** | **100** |

## Appendix 15a: Diagnostic Numeracy Test Results (2005-08 cohort)

| Research Reference Number | Basic Arith-metic | Percen-tages and Ratio | Basic Algebra | Problem -Solving | Significant Figures & Indices | Charts Tables Graphs | Fractions Decimals | TOTAL ( of 77) | % |
|---|---|---|---|---|---|---|---|---|---|
| ED07 | 11 | 10 | 11 | 11 | 11 | 10 | 9 | **73** | **95** |
| **Sharon** | **11** | **7** | **11** | **11** | **11** | **9** | **11** | **71** | **92** |
| ED16 | 9 | 10 | 10 | 10 | 10 | 9 | 10 | **68** | **88** |
| **Suzy** | **10** | **10** | **10** | **10** | **8** | **8** | **10** | **66** | **86** |
| | 11 | 9 | 11 | 10 | 9 | 6 | 10 | **66** | **86** |
| ED14 | 10 | 8 | 10 | 8 | 10 | 8 | 11 | **65** | **84** |
| | 9 | 6 | 11 | 10 | 9 | 10 | 10 | **65** | **84** |
| ED22 | 9 | 8 | 8 | 9 | 10 | 9 | 10 | **63** | **82** |
| | 11 | 7 | 11 | 10 | 7 | 7 | 7 | **60** | **78** |
| | 10 | 9 | 6 | 8 | 9 | 8 | 9 | **59** | **77** |
| | 8 | 9 | 10 | 7 | 8 | 7 | 10 | **59** | **77** |
| | 10 | 8 | 7 | 11 | 8 | 6 | 8 | **58** | **75** |
| ED03 | 11 | 7 | 10 | 8 | 6 | 6 | 8 | **56** | **73** |
| | 6 | 9 | 9 | 10 | 7 | 6 | 9 | **56** | **73** |
| | 8 | 5 | 8 | 10 | 9 | 7 | 7 | **54** | **70** |
| | 8 | 9 | 8 | 7 | 8 | 7 | 7 | **54** | **70** |
| **Dawn** | **7** | **10** | **5** | **7** | **7** | **8** | **8** | **52** | **68** |
| | 8 | 7 | 8 | 8 | 9 | 6 | 5 | **51** | **66** |
| | 6 | 9 | 7 | 8 | 8 | 7 | 6 | **51** | **66** |
| | 8 | 3 | 9 | 8 | 6 | 8 | 8 | **50** | **65** |
| ED11 | 8 | 5 | 8 | 8 | 8 | 7 | 6 | **50** | **65** |
| | 7 | 5 | 8 | 6 | 8 | 8 | 8 | **50** | **65** |
| ED10 | 7 | 9 | 6 | 7 | 5 | 8 | 7 | **49** | **64** |
| | 7 | 7 | 3 | 10 | 7 | 8 | 7 | **49** | **64** |
| | 6 | 6 | 7 | 6 | 8 | 8 | 8 | **49** | **64** |
| | 9 | 3 | 8 | 6 | 6 | 6 | 10 | **48** | **62** |
| | 9 | 4 | 7 | 3 | 8 | 8 | 9 | **48** | **62** |
| | 7 | 6 | 7 | 9 | 6 | 5 | 7 | **47** | **61** |
| | 5 | 7 | 7 | 10 | 5 | 5 | 8 | **47** | **61** |
| ED05 | 6 | 3 | 6 | 7 | 9 | 5 | 8 | **44** | **57** |
| | 7 | 3 | 3 | 6 | 9 | 7 | 5 | **40** | **52** |
| | 0 | 0 | 0 | 0 | 0 | 7 | 0 | **7** | **9** |

**Notes:**

The first column indicates the research study reference number which was allocated when trainees submitted lesson plans or agreed to be interviewed. Where there is no number, this indicates a trainee who took the test but was not forthcoming in providing either lesson plans or interview from this cohort.

**The bold highlighted rows indicate trainees from amongst the seven case studies, whose reference numbers are replaced by their pseudonyms.**

## Appendix 15b: Diagnostic Numeracy Test Results (2006-09 cohort)

| Research Reference Number | Basic Arithmetic | Fractions Decimals | Percentages and Ratio | Basic Algebra | Significant Figures & Indices | Problem Solving | Charts Tables Graphs |
|---|---|---|---|---|---|---|---|
| ED23 | S | NR | S | NR | NR | NR | S |
| **Naomi** | - | - | - | - | - | - | - |
| ED25 | - | - | - | - | - | - | - |
| **Rachael** | **S** | **S** | **S** | **S** | **NR** | **NR** | **S** |
| ED27 | S | E | S | S | S | S | S |
| ED28 | S | S | S | S | NR | S | S |
| ED29 | E | S | S | E | S | S | S |
| ED30 | S | S | S | E | S | S | S |
| ED31 | E | S | S | E | S | S | NR |
| ED32 | S | E | S | S | S | E | S |
| ED33 | - | - | - | - | - | - | - |
| ED34 | S | NR | S | S | S | NR | NR |
| ED35 | NR | S | S | NR | NR | S | NR |
| ED36 | S | E | E | E | E | E | S |
| **Andy** | **S** | **NR** | **NR** | **NR** | **S** | **NR** | **S** |
| ED38 | S | S | S | NR | S | S | NR |
| ED39 | S | S | S | NR | S | S | S |
| ED40 | S | S | E | E | S | E | S |
| ED41 | S | S | S | E | S | E | S |
| ED42 | S | S | NR | S | S | NR | NR |

**Notes**:

The first column indicates the research study reference number which was allocated when trainees submitted lesson plans or agree to be interviewed.

**The bold highlighted rows indicate trainees from amongst the seven case studies, whose reference numbers are replaced by their pseudonyms.**

In 2006, the test results were reported as being one of 'Excellent', 'Satisfactory' or 'Needs Revision' and these are indicated above as E, S or NR.

Blank results indicate that either the result for that trainee was not available that year, or the trainee took the test in a different year and it was not available.

**Appendix 16a: B.Ed Module results for sample group 2005-08 cohort**

| Ref No. | Pseudonym | Y1 | Y2 | Y3 |
|---|---|---|---|---|
| ED01 | Anthea | 46 | 48 | 57 |
| ED02 | Sonia | 51 | 58 | 62 |
| ED03 | Valerie | 51 | 37 | Not available |
| ED04 | Michelle | 53 | 54 | 62 |
| ED05 | Joanna | 51 | 52 | 52 |
| **ED06** | **Suzy** | **62** | **68** | **78** |
| ED07 | Colin | 56 | 74 | 68 |
| ED08 | Livvy | 52 | 55 | 55 |
| ED09 | Jane | 50 | 58 | 55 |
| ED10 | Colette | 52 | 48 | 36 |
| ED11 | Kirsty | 59 | 48 | 66 |
| ED12 | Kerry | 41 | 62 | 48 |
| **ED13** | **Dawn** | **65** | **57** | **80** |
| ED14 | Penny | 60 | 72 | 75 |
| ED15 | Wendy | 54 | 72 | 62 |
| ED16 | Pam | 63 | 52 | 80 |
| ED17 | Lee | 54 | 48 | 67 |
| **ED18** | **Sharon** | **71** | **67** | **68** |
| ED19 | Hayley | 40 | 61 | 52 |
| ED20 | Tania | 44 | 58 | 48 |
| **ED21** | **Victor** | **53** | **55** | **48** |
| ED22 | Kris | 56 | 58 | 48 |

Notes:

**Bold large rows represent the modules marks for the case study trainees.**

Marks given are percentages.

**Appendix 16b: B.Ed Module results for sample group 2006-09 cohort**

| Ref.No. | Pseudonym | Y1 | Y2 | Y3 |
|---|---|---|---|---|
| ED23 | Judith | 52 | 58 | 64 |
| **ED24** | **Naomi** | **40** | **45** | **40** |
| ED25 | Elizabeth | 49 | 58 | 65 |
| **ED26** | **Rachael** | **60** | **58** | **65** |
| ED27 | Dawn | 64 | 78 | 76 |
| ED28 | Helen | 51 | 82 | 42 |
| ED29 | Clare | 47 | 62 | 54 |
| ED30 | Sara | 55 | 66 | 54 |
| ED31 | Lauretta | 50 | 48 | 52 |
| ED32 | Phillippa | 66 | 62 | 62 |
| ED33 | Carl | 40 | 48 | 52 |
| ED34 | Tracy | 45 | 54 | 40 |
| ED35 | Ellie | 48 | 40 | 45 |
| ED36 | Mandy | 63 | 78 | 72 |
| **ED37** | **Andy** | **48** | **57** | **62** |
| ED38 | Anna | 49 | 48 | 42 |
| ED39 | Jane | 51 | 42 | 55 |
| ED40 | Lianne | 57 | 68 | 67 |

Notes:

**Bold large rows represent the modules marks for the case study trainees.**

Marks given are percentages.

**Appendix 17a: Number of lesson plans provided by trainees in 2005-08 cohort, listed by number in each school year group**

| Year groups | Number of lessons |
|---|---|
| Year 4 | 79 |
| Year 3 | 57 |
| Year 2 | 57 |
| Reception | 49 |
| Year 2/Year 3 | 43 |
| Year 3/Year 4 | 41 |
| Reception/Year 1 | 34 |
| Year 5 | 30 |
| Year 4/Year 5 | 28 |
| Year 6 | 24 |
| Year 1/Year 2 | 19 |

**Appendix 17b: Number of lesson plans provided by trainees in 2005-08 cohort, listed by number for each mathematics topic**

| Topic | Number of lessons |
|---|---|
| Addition and Subtraction | 66 |
| Multiplication | 52 |
| 2D and 3D shape | 42 |
| Fractions/Decimals/Percentages | 40 |
| Measures | 34 |
| Word problems | 34 |
| Sequences | 26 |
| Counting and rounding | 26 |
| Division | 24 |
| Data handling | 19 |
| Angles | 17 |
| Time | 16 |
| Area/perimeter | 15 |
| Number bonds | 13 |
| Symmetry | 11 |
| Compasses | 9 |
| One more/one less | 9 |
| Calculations | 4 |
| Probability | 2 |
| Algebra | 1 |
| Coordinates | 1 |

**Appendix 18a: Number of lesson plans provided by trainees in 2006-09 cohort, listed by number in each school year group**

| Year groups | Number of lessons |
|---|---|
| Year 3 | 113 |
| Year 3/Year 4 | 95 |
| Year 5 | 33 |
| Year 4 | 26 |
| Year 1/Year 2 | 20 |
| Year 1 | 12 |
| Year 6 | 3 |

**Appendix 18b: Number of lesson plans provided by trainees in 2006-09 cohort, listed by number for each mathematics topic**

| Topic | Number of lessons |
|---|---|
| Addition/Subtraction | 58 |
| Fractions | 40 |
| Multiplication /Division | 33 |
| 2D & 3D shape | 30 |
| Data Handling | 28 |
| Time | 20 |
| Word problems | 12 |
| Capacity | 12 |
| Mass | 10 |
| Money | 10 |
| Symmetry | 9 |
| Problem solving | 6 |
| Sequences | 6 |
| Directions | 5 |
| Length | 5 |
| Coordinates | 3 |
| Area | 3 |
| Place Value | 3 |
| Reading scales | 2 |
| Angles | 2 |
| Decimals | 2 |
| Negatives | 1 |
| Inverse Operations | 1 |
| Measures | 1 |